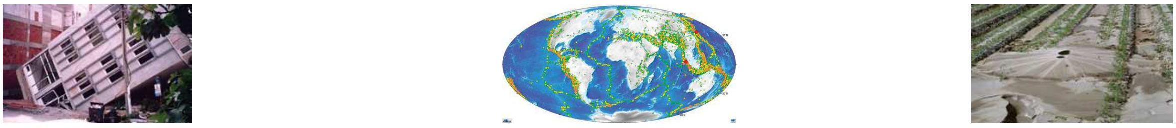

# GEOMECHANICS AND ADVANCED GEOTECHNICS

## Engineering seismology and soil dynamics.

Jean-Francois Semblat

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et Applications Industrielles (IMSIA), Palaiseau
`jean-francois.semblat@ensta-paris.fr`

January, 9, 2023

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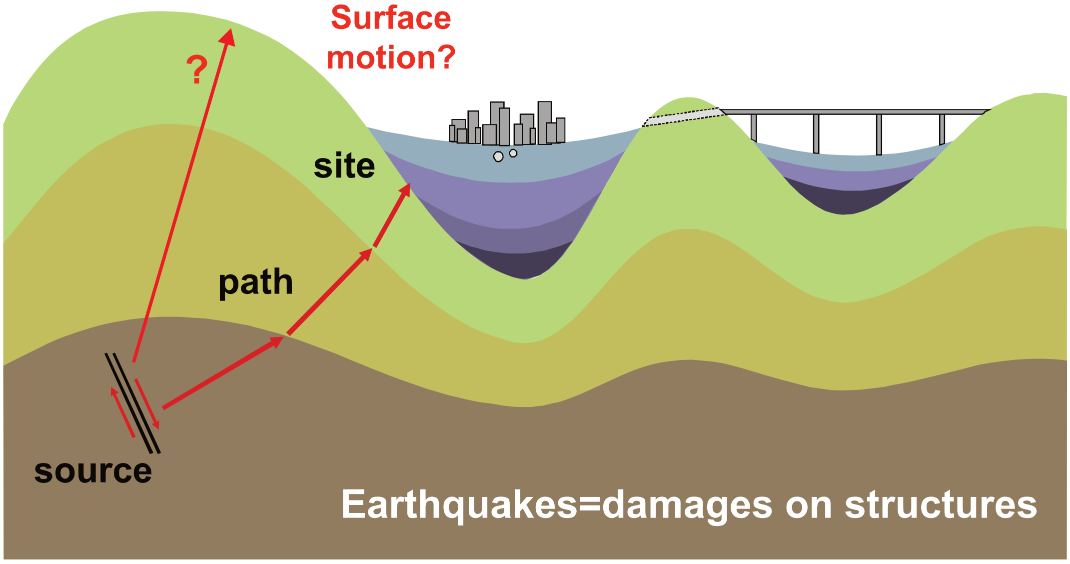
# Introduction

## Engineering seismology and soil dynamics

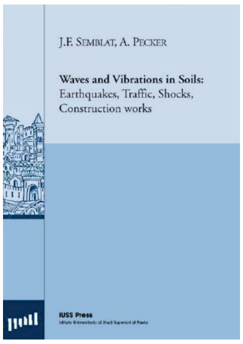
- From source to structures through site:



# Introduction

## Engineering seismology and soil dynamics

- References:
  - *Waves and Vibrations in Soils: Earthquakes, Traffic, Shocks, Construction Works*, Semblat J-F, Pecker A., IUSS Press, 2009.

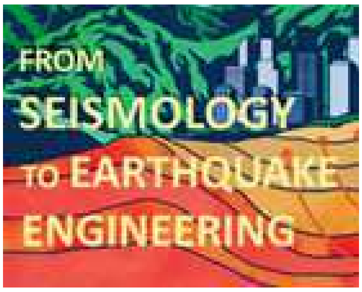


J.F. SEMBLAT, A. PECKER

Waves and Vibrations in Soils:
Earthquakes, Traffic, Shocks,
Construction works

IUSS Press

  - MOOC *"From seismology to earthquake engineering"*:
    Teaser: youtu.be/V6mnVBszk0E
    Enroll: www.coursera.org



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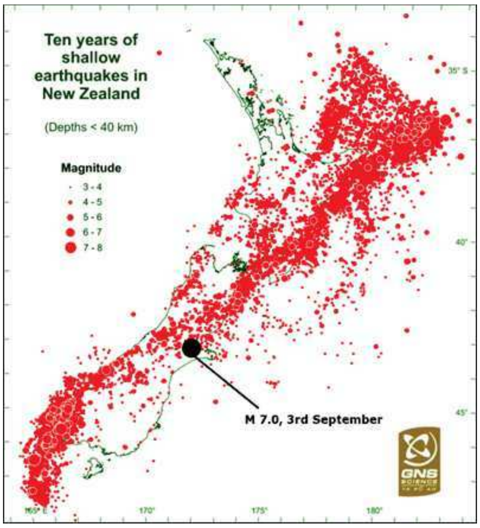
# Contents

# Seismicity

## Origin of earthquakes: Canterbury fault

- Map and fault (GNS)



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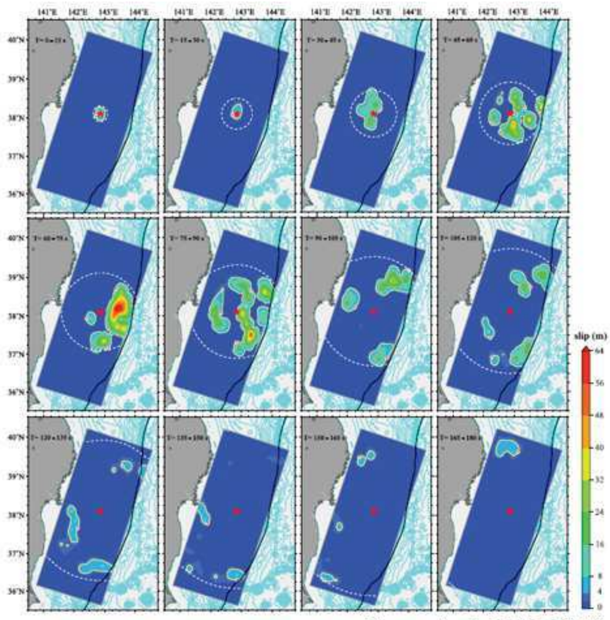
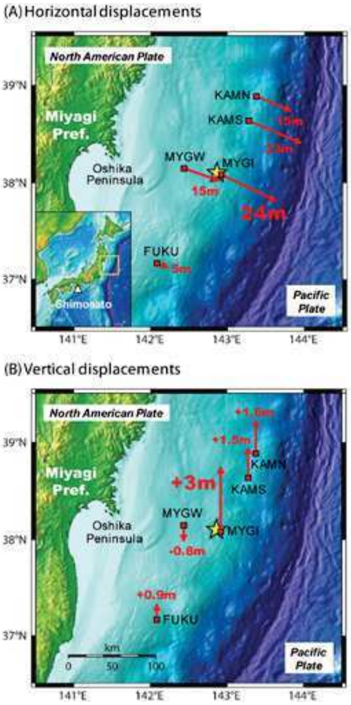
# Seismicity

## Origin of earthquakes: Tohoku quake

- Displacements at the fault & at the surface



Shao et al., EPS 2011

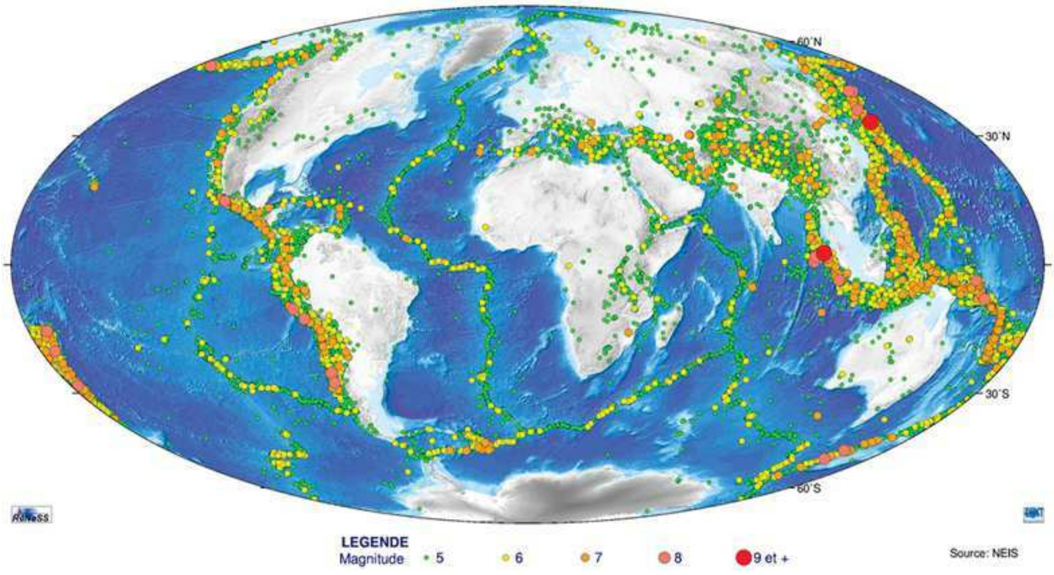


Sato et al., Science 2011

# Seismicity

## World seismicity

- Earthquakes in the world from 1973 to 2012



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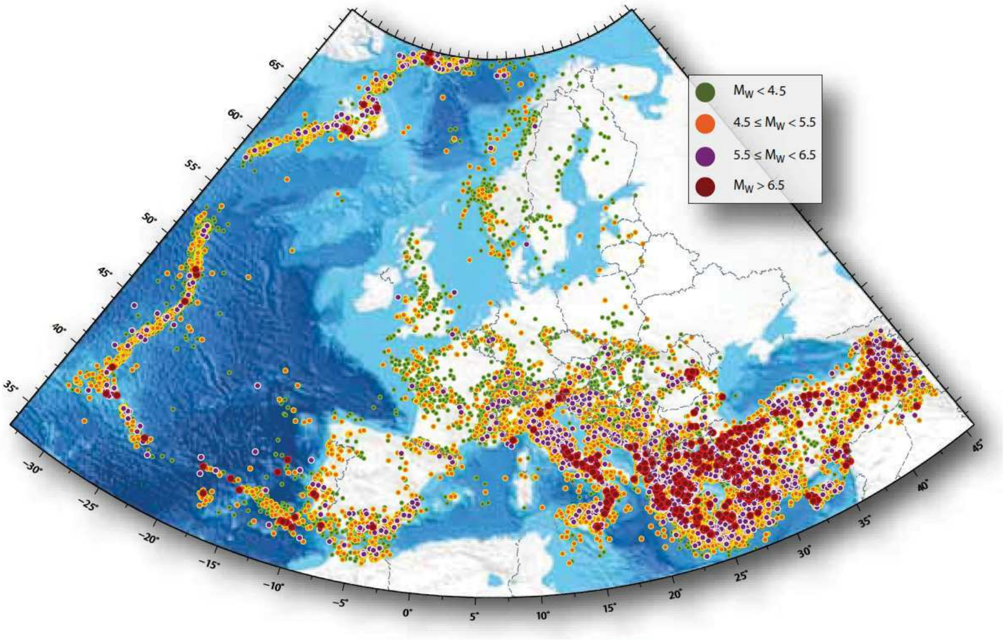
# Seismicity

## Seismicity of Europe

- Earthquakes in Europe for the period 1000-2007 with moment magnitudes $M_w \geq 3.5$ (SHARE).



# Seismicity

## Observation in Japan

JF Semblat

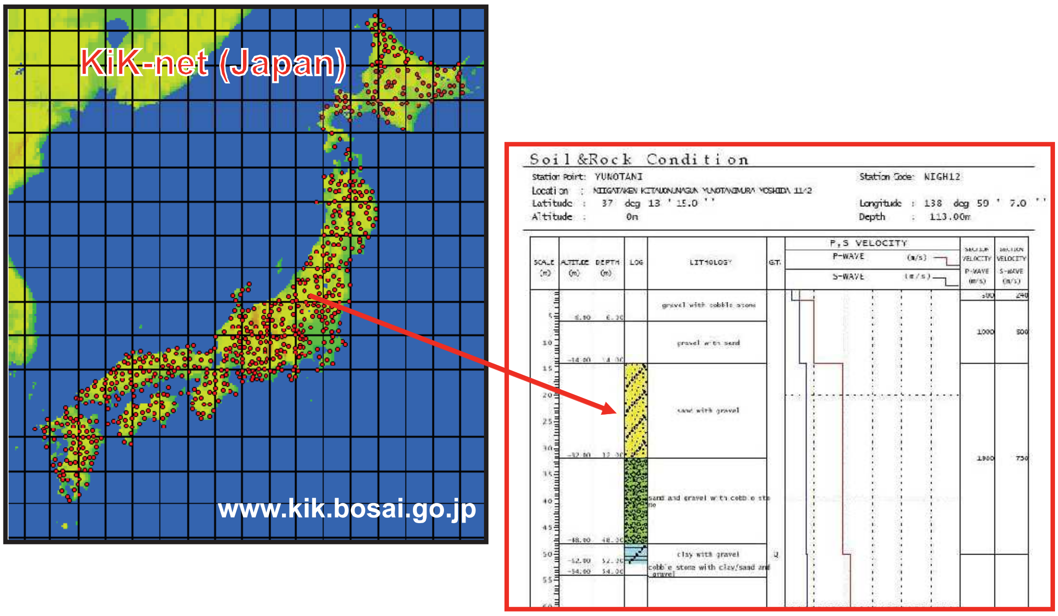
Seismicity

Seismic waves
- Elastodyn.
- Amplification
- Basin effects

Attenuation
- Viscoelastic approxim.
- Damped waves

Perspectives

- Kik-net: observations in Japan



Soil &Rock Condition

Station Point: YUNOTANI
Station Code: NIGH12
Location : NIIGATAKEN KITAUONUMAGUN YUNOTANIMURA YOSHIDA 11-2
Latitude : 37 deg 13 ' 15.0 ''
Longitude : 138 deg 50 ' 7.0 ''
Altitude : 0m
Depth : 113.00m

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# Contents

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Seismicity

Seismic waves
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- Damped waves

Perspectives

# Body waves

## Wave equations

- Uncoupled elastic wave equations:

$$\begin{cases} \Delta(\phi) = \dfrac{1}{V_P^2}\dfrac{\partial^2\phi}{\partial t^2} \text{ with } V_P = \sqrt{\dfrac{\lambda+2\mu}{\rho}} \\ \Delta(\underline{\psi}) = \dfrac{1}{V_S^2}\dfrac{\partial^2\underline{\psi}}{\partial t^2} \text{ with } V_S = \sqrt{\dfrac{\mu}{\rho}} \end{cases}$$

- Typical velocity values:

| Material | $V_S$ (m/s) | $V_P$ (m/s) |
|---|---|---|
| Sand | 200-400 | 400-800 |
| Clay | 100-200 | 1500 |
| Rock | >800 | >2000 |

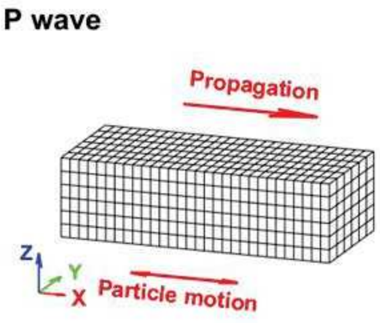
$$\frac{V_P}{V_S} = \sqrt{\frac{2-2\nu}{1-2\nu}}$$

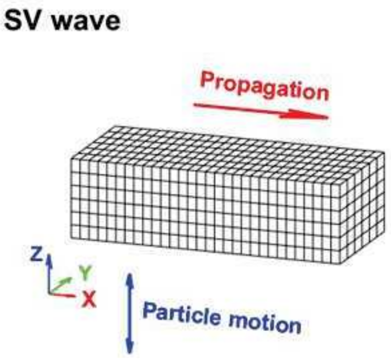
---

# Body waves

## Wave types

- Disregarding the boundary conditions, the wave equations define body waves as follows (Poisson, 1828):
  - primary or pressure waves (P-waves)
  - secondary or shear waves (S-waves)

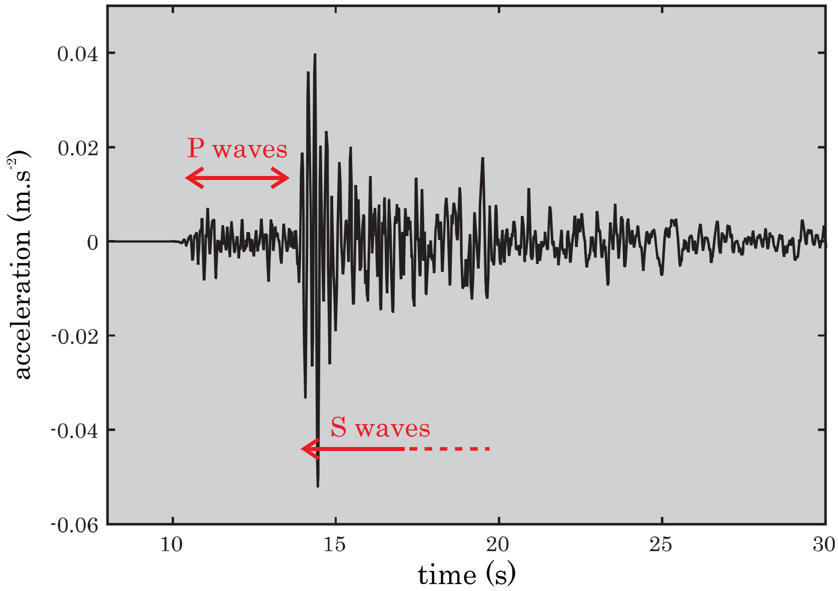




# Body waves

## Wave types

- In 1897, Richard D. Oldham identifies the *different wave types* in a seismic recording
- Ex.: earthquake recording in the city of Nice (CEREMA)



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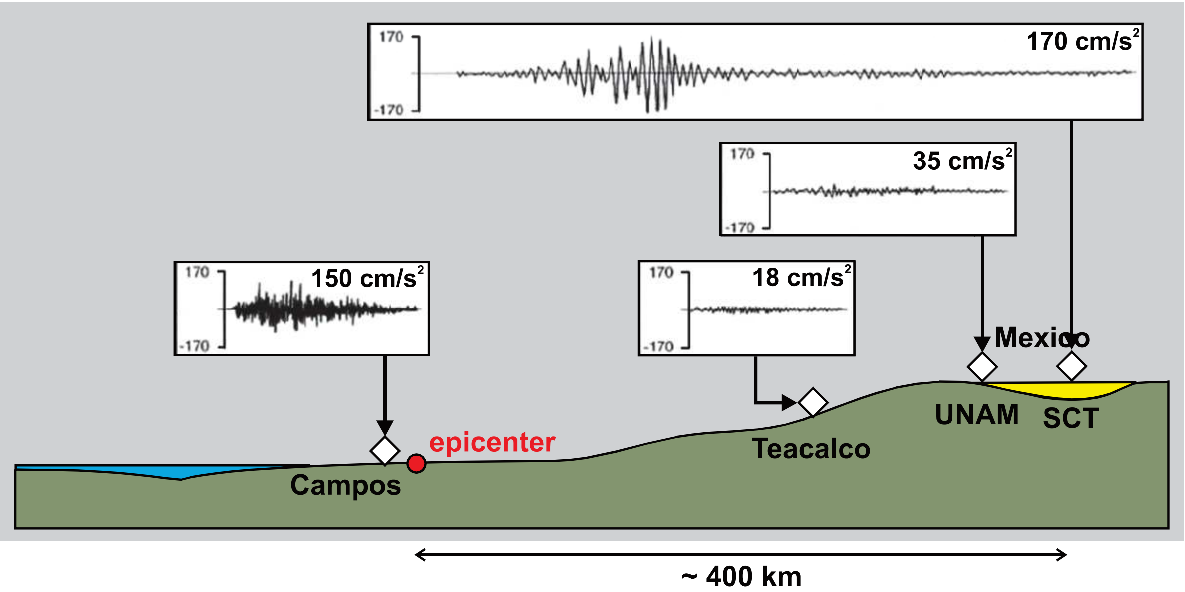
# Contents

# Amplification

## Evidences: Mexico (85)

- Mexico-Michoacan earthquake(M8.0, sept., 19, 1985)



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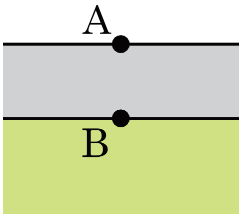
# Amplification

## Analysis: single layer case

- Elastic layer on an elastic half-space, harmonic SH-wave: spectral ratio $A/B$:



$$T(\omega) = \frac{u_A}{u_B} = \frac{2A_1}{A_2 + A_2'} \Rightarrow \boxed{|T(\omega)| = \frac{1}{\cos\left(k_{z_1} h\right)}}$$

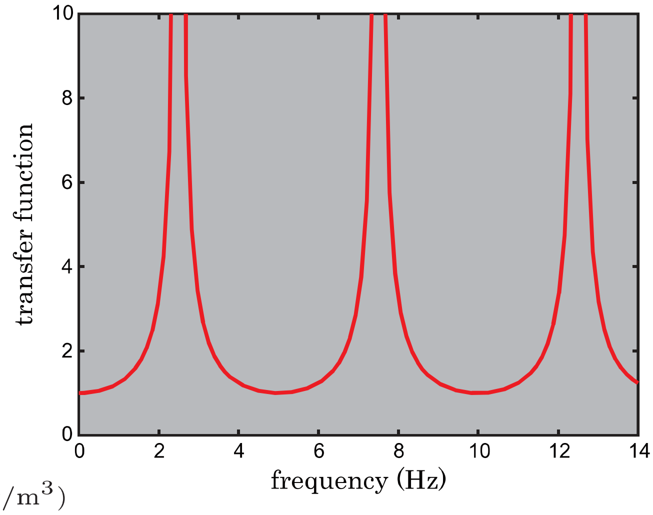
independent of $\mu_2$!

- $T(\omega)$ infinite for:

$$\boxed{\omega_n = \frac{(2n-1)\pi}{2} \frac{V_{S_1}}{h \cos\theta_1}}$$

$$\omega_1 = 2\pi f_1 \Rightarrow$$

$$\boxed{f_1 = \frac{V_{S_1}}{4h} \frac{1}{\cos\theta_1}}$$



($h$=20m, $V_{S_1}$=200m/s, $\rho$=2000kg/m$^3$)

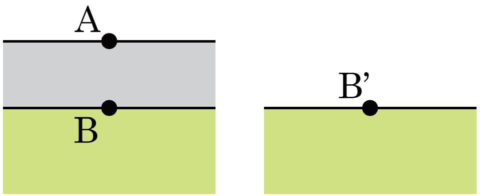
# Amplification

## Analysis: single layer case

- Alternative spectral ratio: motions at $A$ and $B'$
- $B'$: hypothetical location at interface without layer!

$$T^*(\omega) = \frac{u_A}{u_{B'}} = \frac{2A_1}{2A_2} \Rightarrow$$

$$\boxed{T^*(\omega) = \frac{1}{\cos(k_{z_1}h) + i\bar{\chi}\sin(k_{z_1}h)}}$$



with: $\bar{\chi} = \sqrt{\frac{\mu_1\rho_1}{\mu_2\rho_2}\frac{\cos\theta_1}{\cos\theta_2}}$

- $T^*(\omega) \neq T(\omega)$ since:
  - It depends on the properties of both media ($\bar{\chi}$)
  - The peaks have a finite amplitude $= 1/\bar{\chi}$: energy partially absorbed by the reflected wave $A_2'$ (radiative damping). It is not the case when $\bar{\chi}$ is large.

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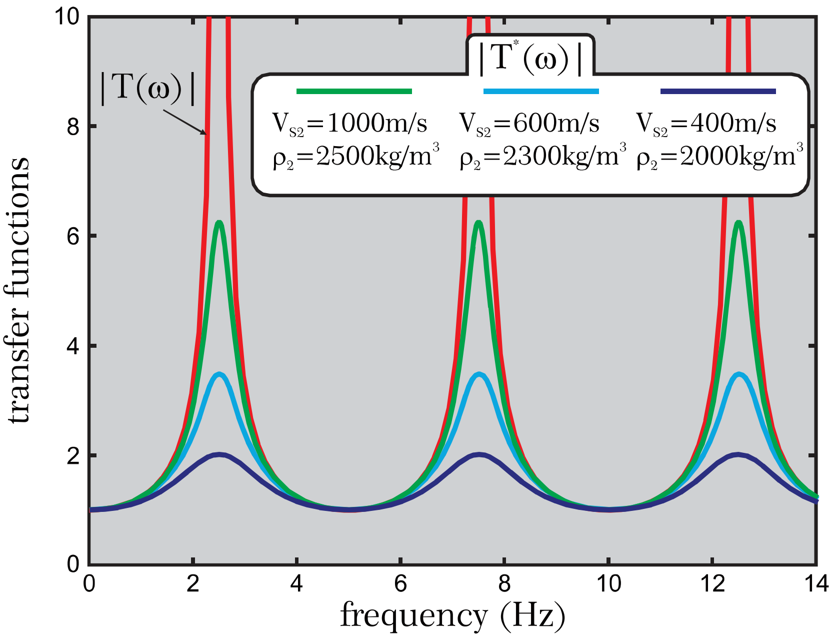
# Amplification

## Analysis: single layer case

- Amplification for various velocity contrasts



# Contents

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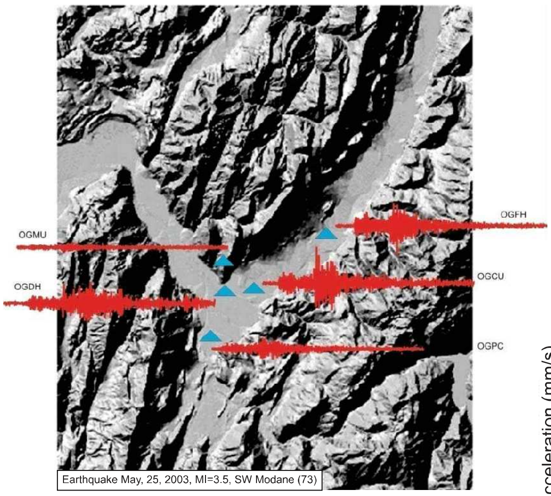
# Basin effects

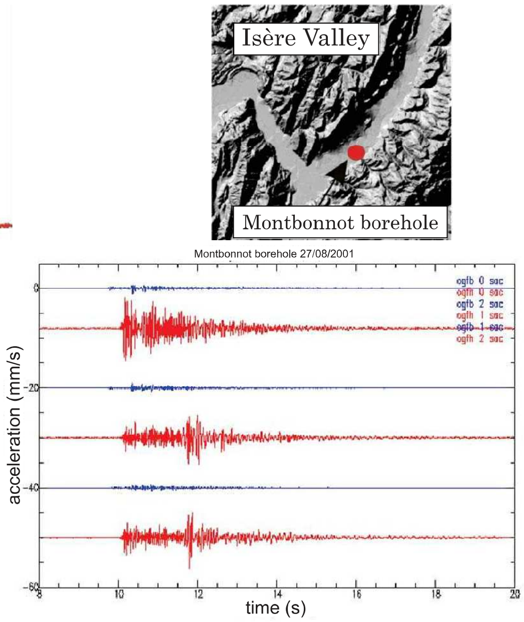
## Alpine valleys

- Deep alpine valley (Grenoble, France)

Site effects in Grenoble



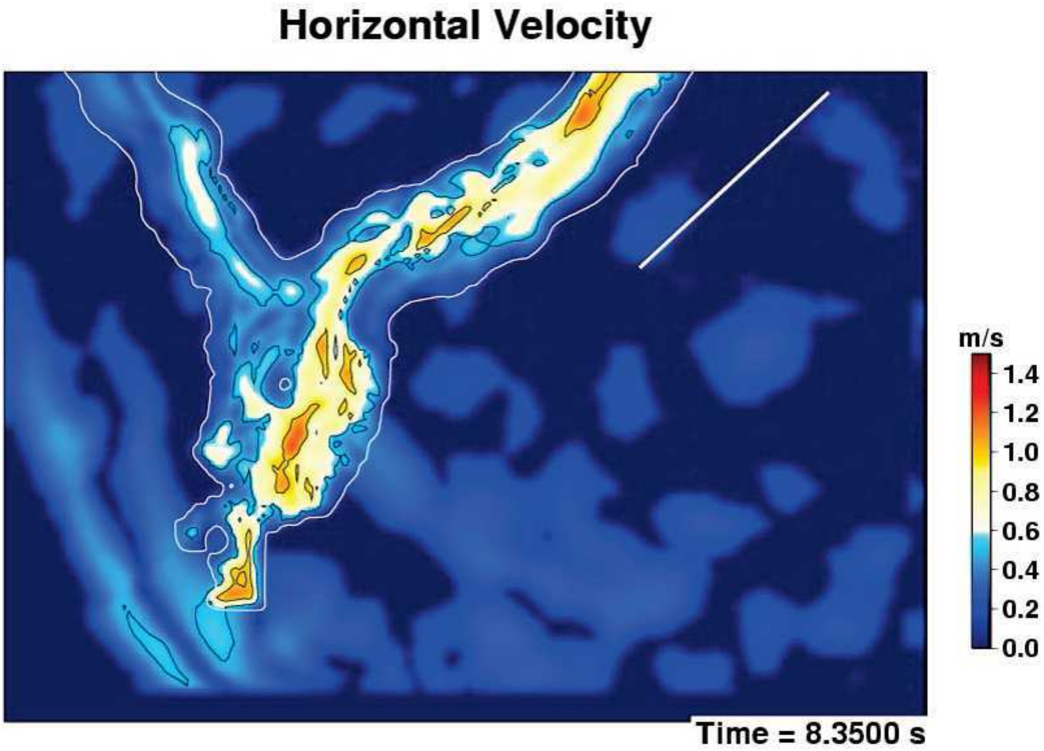
Montbonnot borehole



# Basin effects

## Alpine valleys

- Deep alpine valley (Grenoble, France): SEM simulations



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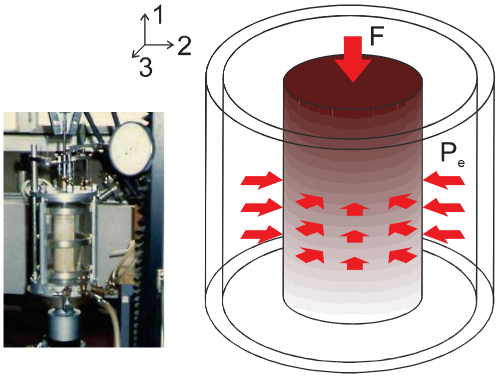
# Contents

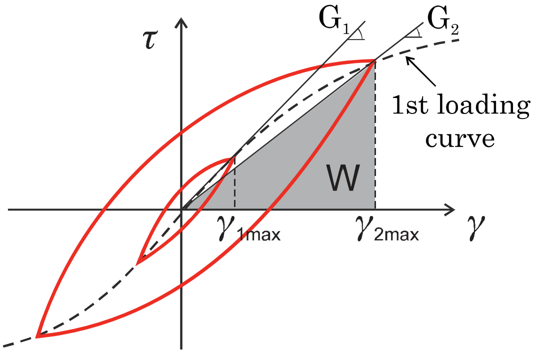
# Anelasticity

## Cyclic loadings on soils

- Cyclic triaxial test



Hysteresis loops



- Damping: $\boxed{\beta = \dfrac{\Delta E}{4\pi W}}$
  - $\Delta E$: energy dissipated in a cycle (area of the hysteresis loop)
  - $W$: elastic energy stored during 1/4 cycle

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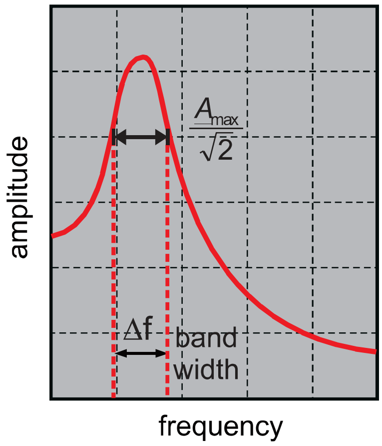
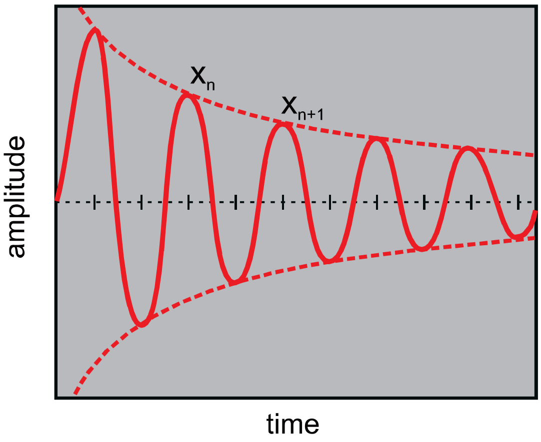
# Anelasticity

## Vibration experiments

- Resonant column: direct estimation of damping:





- In freq. domain: $\beta = \dfrac{\Delta f}{2 f_0}$
- In time domain: $\delta = 2\pi\beta = \log \dfrac{x_{n+1}}{x_n}$

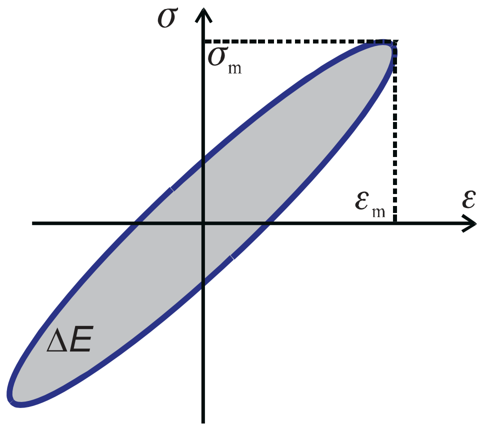
# Viscoelastic approximation

## Dealing with energy dissipation

- Needs: damping for continuous models
- Framework: linear viscoelasticity
- Behaviour depends on **loading history**: $\underline{\underline{\sigma}} = f(\underline{\underline{\varepsilon}}, \underline{\underline{\dot{\varepsilon}}})$

$\sigma = f(\varepsilon, \dot{\varepsilon})$ (longitudinal) or $\tau = f(\gamma, \dot{\gamma})$ (torsion), etc.

- Phase shift between stress and strain. For $\hat{\varepsilon}(\omega) = \varepsilon_m \exp(i\omega t)$, the stress-strain curve is:



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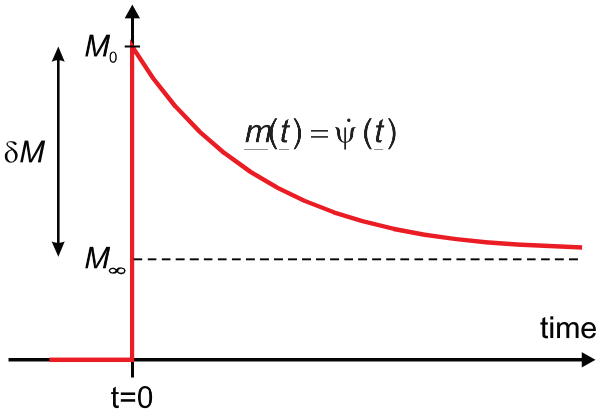
# Viscoelastic approximation

## Viscoelastic modulus $m(t)$

- The constitutive law may be written:

$$\sigma = \int_{-\infty}^{+\infty} m(t-\tau)\varepsilon(\tau)d\tau = m(t) * \varepsilon(t)$$

- $m(t)$: response in stress to a Dirac strain $\varepsilon(t) = \delta(t)$



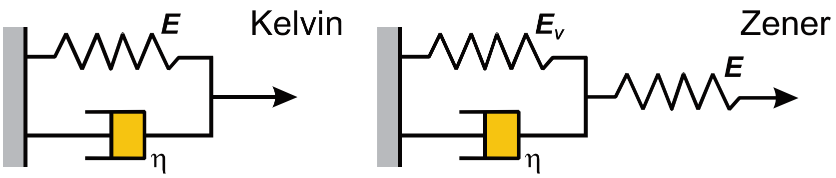
$M_0$: instantaneous modulus, $M_\infty$: relaxed modulus

# Rheological models

## Complex modulus $M(\omega)$

- In the frequency domain, the constitutive law becomes:

$$\boxed{\sigma(t) = m(t) * \varepsilon(t) \quad \rightarrow \quad \hat{\sigma}(\omega) = M(\omega)\hat{\varepsilon}(\omega)}$$



- Kelvin-Voigt: $\sigma = \sigma_e + \sigma_v = E\varepsilon + \eta\dot{\varepsilon}$

  In freq.: $\hat{\sigma} = (E + i\omega\eta)\hat{\varepsilon} = M(\omega)\hat{\varepsilon}$

- Zener: $\hat{\sigma} = \dfrac{E(E_v + i\omega\eta)}{E + E_v + i\omega\eta}\hat{\varepsilon} = \dfrac{1 + i\omega\tau_\varepsilon}{1 + i\omega\tau_\sigma} M_\infty \hat{\varepsilon}$

- Attenuation: $\boxed{Q^{-1}(\omega) = \dfrac{M_I(\omega)}{M_R(\omega)} \simeq 2\beta(\omega)}$ $\quad Q_K^{-1}(\omega) = \dfrac{\omega\eta}{E}$

---

# Contents

1 Seismicity

2 Seismic wave amplification
- Elastodynamics
- Amplification of seismic waves
- Basin effects

3 Attenuation of seismic waves
- Viscoelastic approximation
- Wave propagation in viscoelastic media

4 Perspectives

# Wave propagation in viscoelastic media

## Wave equation for longitudinal waves

- Equation of motion (freq.domain):

$$\frac{\partial \hat{\sigma}(x,\omega)}{\partial x} = \rho \frac{\partial^2 \hat{u}(x,\omega)}{\partial t^2} = -\rho\omega^2 \hat{u}(x,\omega)$$

- Viscoelastic behaviour: $\hat{\sigma}(x,\omega) = M(\omega)\hat{\varepsilon}(x,\omega)$

$$\frac{\partial \hat{\sigma}(x,\omega)}{\partial x} = M(\omega)\frac{\partial^2 \hat{u}(x,\omega)}{\partial x^2} = -\rho\omega^2 \hat{u}(x,\omega)$$

- Wave equation in the damped case:

$$\boxed{\frac{\partial^2 \hat{u}(x,\omega)}{\partial x^2} = -\frac{\rho\omega^2}{M(\omega)}\hat{u}(x,\omega)}$$

---

# Wave propagation in viscoelastic media

## Solution, complex wavenumber

- Solution: longitudinal damped wave

$$\hat{u}(x,\omega) = U_0(\omega)\exp[-i\hat{k}(\omega)x]$$

where $\hat{k}(\omega)$ is the complex wavenumber such as:

$$\boxed{\hat{k}^2(\omega) = \frac{\rho\omega^2}{M(\omega)}}$$

- Denoting $\hat{k}(\omega) = k(\omega) - i\alpha(\omega)$, the solution reads:

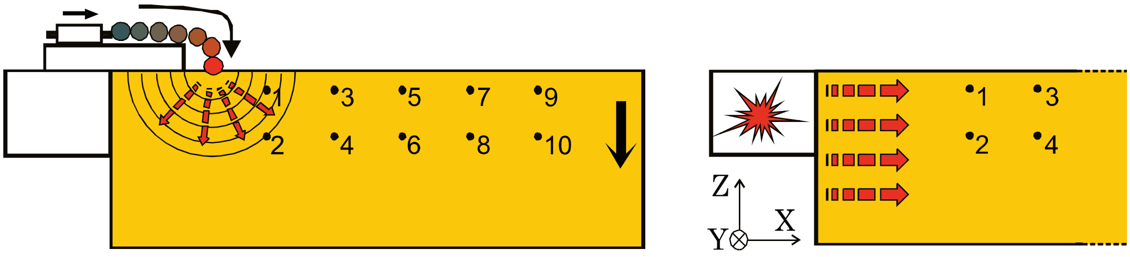
$$\boxed{\hat{u}(x,\omega) = U_0(\omega)\exp\left[-ik(\omega)x\right]\exp\left[-\alpha(\omega)x\right]}$$

propagation attenuation

# Wave propagation in viscoelastic media

## Examples

- Centrifuge experiments:

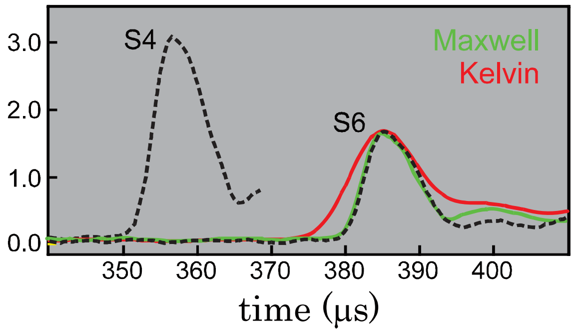


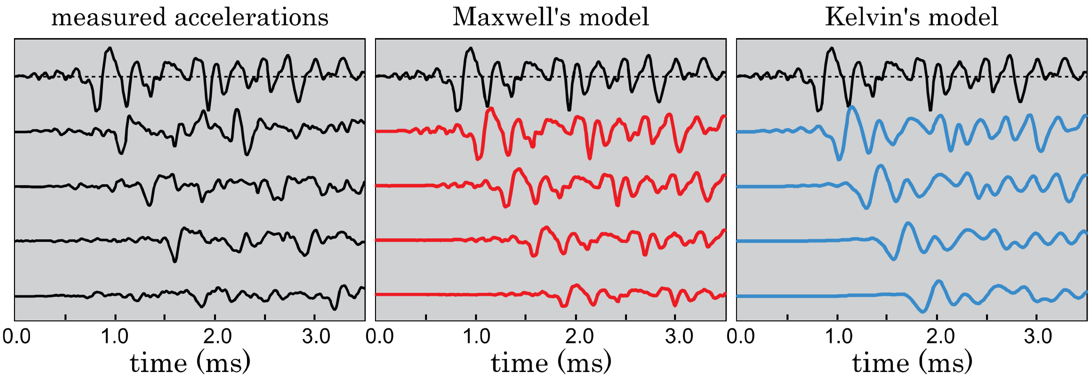
- Solution 1: $\hat{u}(r,\omega) = \frac{A}{r}\left(-\frac{1}{r} - i\hat{k}(\omega)\right)\exp\left[-i\hat{k}(\omega)r\right]$
- Solution 2: $\hat{u}(x,\omega) = A\exp\left[-i\hat{k}(\omega)x\right]$

# Wave propagation in viscoelastic media

## Examples

- Centrifuge experiments:




measured accelerations

Maxwell's model

Kelvin's model

# Contents

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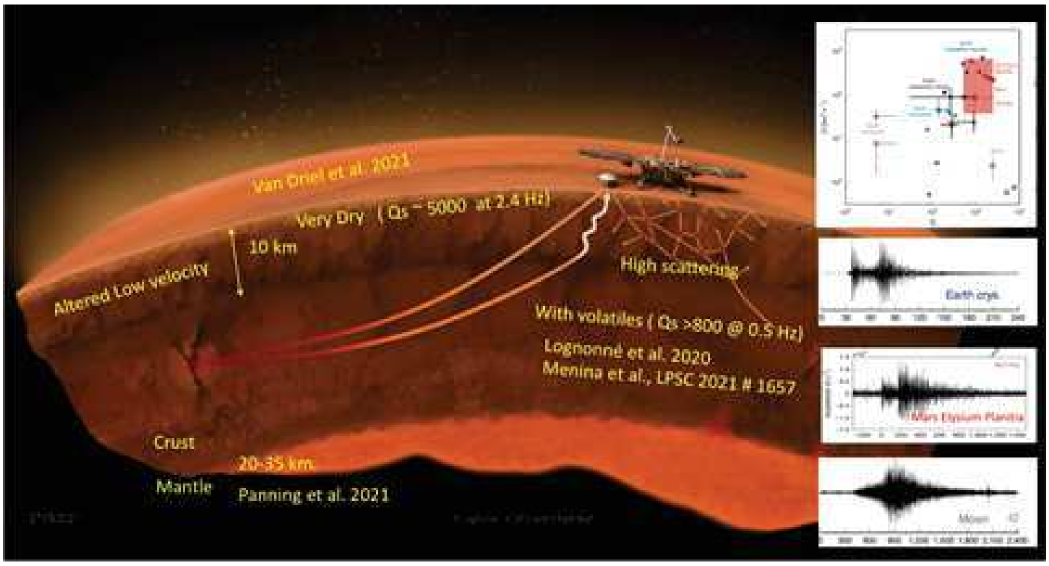
# Marsquakes!

## Identification from Mars structure

- Observations from the Insight mission
- Mars crustal structure in Elysium Planitia

  (Lognonné et al. 2020, Knapmeyer-Endrun et al., 2021)



`MOOC "From Seismology to Earthquake Engineering" (Coursera)`

# Strong ground motions

## Liquefaction

- Strong motion: the Tohoku event

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# References

- Aki K., Richards P.G. (1980), *Quantitative Seismology*, Freeman & Co
- Chaillat S., Bonnet M., Semblat J.F. (2009). A new fast multi-domain BEM to model seismic wave propagation and amplification in 3D geological structures, *Geophys. Journal Int.*, 177(2): 509-531.
- Delépine N., Bonnet G., Lenti L., Semblat J.F. (2009). Nonlinear viscoelastic wave propagation: an extension of Nearly Constant Attenuation models, *Journal of Eng. Mechanics (ASCE)*, 135(11): 1305-1314.
- Kramer S.L. (1996), *Geotechnical earthquake engineering*, Prentice-Hall.
- Régnier J., L.F. Bonilla, E. Bertrand, J.F. Semblat (2014). Influence of the Vs profiles beyond 30 m depth on linear site effects: assessment from the KiK-net data, *Bull. Seism. Soc. Am.*, 104(5): 2337-2348.
- Santisi d'Avila M.P., Lenti L., Semblat J.F. (2012). Modeling strong seismic motion: 3D loading path vs wavefield polarization, *Geophys. Jal Int.*, 190: 1607-1624.
- Semblat J.F., Pecker A. (2009). *Waves and vibrations in soils: Earthquakes, Vibrations, Shocks, Construction works*, IUSS Press, Pavia, Italy, 499 p.
- Semblat J.F., Kham M., Parara E., Bard P.Y., Pitilakis K., Makra K., Raptakis D. (2005). Site effects: basin geometry vs soil layering, *Soil Dyn. & Earthquake Eng.*, 25(7-10): 529-538.