

Urban social dynamics

Statistical physics and complex systems tools for the policy-maker

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Statistical Physics

From micro to macro

Statistical physics: tools and concepts for explaining a 'macro' level from the properties of the 'micro' level composed of a large number of interacting elements

Allows to derive the laws of thermodynamics

Key concepts: energy, entropy

Equilibrium: balance between order (minimization of energy) and disorder (maximization of entropy)

Modern statistical physics:

- study of highly heterogeneous ('disordered') systems
- study of the dynamics of systems far from equilibrium
- studies outside the traditional fields of physics:

biology; economics & social sciences; Data Science.

Statistical Physics

From micro to macro

Statistical physics: tools and concepts for explaining a 'macro' level from the properties of the 'micro' level (explaining 'emergence').

Micro level
(elementary units)

Interactions

Macro level
(collective properties)

magnetic moments
(spins)

interactions

thermodynamics
ferromagnetism

agents' preferences

social influences
("externalities")

market
equilibrium price

neuron (activation
mechanism)

synaptic weights

psychophysics
associative memory

Statistical Physics

Links

Conceptual links

- ▶ Statistical physics
 - ↔ Information theory (Shannon), Statistical inference
 - ↔ Game theory
- ▶ Statistical physics & dynamical systems ↔ complex systems

Specific formal links - for instance:

- ▶ neuroscience: Memory models (Hopfield 82) ↔ physics: Ising Spin glasses ↔ social science: Coalitions formation (Axelrod, 84)
- ▶ Binary choices under social influence (Schelling 70's) ↔ Random Field Ising model (RFIM, Imry & Ma 1975)
- ▶ Social segregation (Schelling 70's) ↔ "Spin 1" models, surface tension problems

Analysis and Modelling

Methodology

" model " :

- ▶ " a precise and economical statement of a set of relationships that are sufficient to produce the phenomenon in question, or
- ▶ an actual biological, or mechanical, or social system that embodies the relationships in an especially transparent way"

" Most of the models used in the social sciences are families rather than individual models "

" ... a family of related models that differ in some characteristics but share some essential features"

T. C. Schelling (Pr. of Political Economy, Harvard Univ.)

Analysis and Modelling

Methodology

- ▶ Identification of typical properties (stylized facts - e.g. scaling laws)
- ▶ "Phase diagrams": in the space of parameters, determination of the boundaries between domains with qualitatively different behaviours. Boundaries: correspond to phase transitions, bifurcations
- ▶ Analysis of typical and optimal behaviours/performances
- ▶ Confrontation with empirical data,
qualitatively: reproducing stylized facts
quantitatively: data driven modelling
- ▶ Tools:
mathematical modelling (ordinary and partial differential equations;
probabilistic discrete models; game theory;...);
numerical simulations ("agent based models").

Complex systems in urban studies

Self-organization versus top-down planning?

- ▶ Scaling laws (cf. Lecture Marc Barthelemy)
- ▶ Networks (cf. Lecture Marc Barthelemy)
- ▶ Crime patterns [▶ skip](#)
- ▶ Social segregation

Complex systems in urban studies

Crime patterns

- ▶ hot spots of crime activities, eg burglaries
- ▶ gang patterns
- ▶ riots



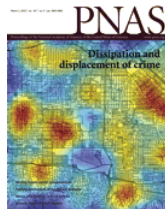
Data driven modelling, partial differential equations and/or agent based models

Complex systems in urban studies

Crime patterns: predictive policing

- UCL, London, T Davies and S. Johnson:
 - ▶ road structure vs. burglary risk

- UCLA: cooperation between mathematicians (A. Bertozzi, M. Short), anthropologists (J. Brantingham), criminologists (G E Tita), and the police of Los Angeles (LAPD) and of other cities
 - ▶ hot spots, gangs, predictive policing



<http://science360.gov/obj/video/f73df48e-c727-4771-a83b-91c05e8aaf01/science-behind-news-predictive-policing> ("Science Behind the News: Predictive Policing", Anne Thompson, correspondent NBC Learn. NBCUniversal Media. 22 Feb. 2013)

Complex systems in urban studies

Crime patterns: predictive policing

mai 2015

LA VIGIE : LE MEILLEUR DU WEB

À LIRE SUR 20MINUTES.FR

21/05/2015 à 10h30

La gendarmerie a un nouveau logiciel pour prédire les délits

Signalé par

[Camille Polloni](#)

« Empêcher que les faits ne se réalisent », c'est l'ambition d'un nouveau logiciel prédictif expérimenté par la gendarmerie nationale pour anticiper les grandes tendances de la délinquance sur le territoire. Déjà [testé en Bavière](#) ou [en Suisse](#), et [utilisé en Californie](#), ce type de logiciel était encore inédit en France.

L'idée est d'analyser certaines catégories de délits fréquents – les cambriolages, les vols, les trafics de stupéfiants ou encore les agressions sexuelles – s'étant produits les cinq dernières années, pour tenter d'en tirer des régularités et de prévoir où et quand ils pourraient se renouveler dans les prochains mois.

Ce « lissage exponentiel » est traité par les chefs de service. « A eux ensuite d'adapter leurs moyens et d'exploiter au mieux ces renseignements criminels dans leurs zones », écrit 20 Minutes. Par exemple en augmentant le nombre de patrouilles aux abords des

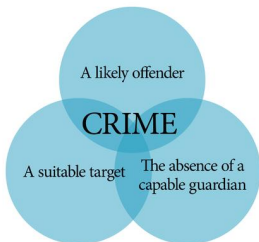
commerces.

[Lire sur 20minutes.fr](#)

Complex systems in urban studies

Crime patterns: from social theory to modeling

ROUTINE ACTIVITY THEORY



Physical convergence in time and space

Lawrence Cohen and Marcus Felson,
Social Change and Crime Rate Trends:
A Routine Activity Approach
American Sociological Review, 1979

modelling:
agents behaviour \leftrightarrow spatial field

Crime patterns

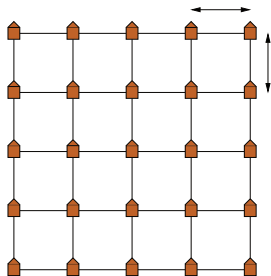
The Short et al. model

[M.B. Short, M.R. D'Orsogna, V.B. Pasour, G.E. Tita, P.J. Brantingham, A.L. Bertozzi, L.B. Chayes.

(2008) A statistical model of criminal behavior. *M3AS* **18**:1249-1267]

Each house is described by its lattice site $s = (i, j)$ and a quantity $A_s(t)$ (attractiveness).

$$A_s(t) = A_s^0 + B_s(t) > 0$$



Probability a burglar commits a burglary:

$$p_s(t) = 1 - e^{-A_s(t)\delta t}$$

During each time interval δt , burglars perform exactly one of the following two tasks:

- 1 Burgle the home at which they are currently located, or
- 2 move to one of the adjacent homes (biased towards high $A_s(t)$).

When a house is burgled:

- ▶ The corresponding burglar is removed from the lattice.
- ▶ B_s is increased by a quantity θ , then decays over time.

Near-repeat victimisation: $B_s(t)$ spreads to its neighbours.

Wandering burglars:

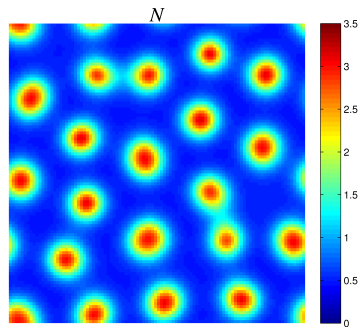
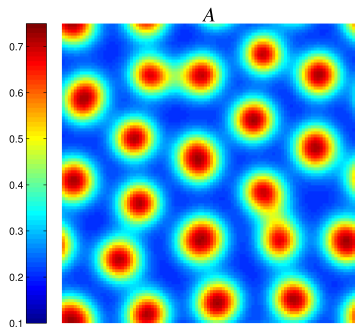
- ▶ Burglars come from sites they did not burgle in the previous time step
- ▶ New burglars are generated at each site at a rate Γ

From the agent based model, continuous time and space limits \rightarrow
Partial Differential Equations (PDEs)

What do the solutions look like?

After 100 (nondimensional) time units
(left: *attractiveness*)

(right: *density of burglars*)



periodic boundary conditions & initial conditions slightly perturbed from the uniform equilibrium state [A.B. Pitcher (2010)]

Complex systems in urban studies

Crime patterns: riots

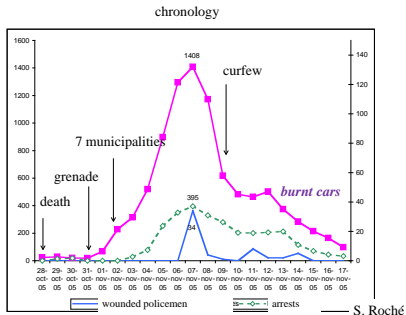
Analysis and modelling of riots patterns

- 2011 London riots: Shane Johnson, UCL, London
- 2005 French riots: collaboration S. Roché (criminologist, Grenoble), M. Gordon (physicist, Grenoble), and at CAMS: L. Bonnasse-Gahot (computer scientist), H. Berestycki (maths), JPN (current work, paper in preparation)

Complex systems in urban studies

Crime patterns: riots

2005 French riots



burnt cars vs time (S. Roché)

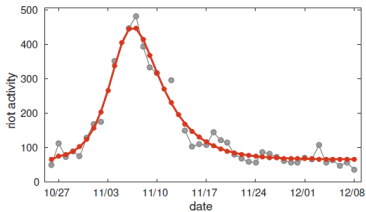


cover of the events by CNN

Complex systems in urban studies

Crime patterns: riots

Epidemic model, contagion

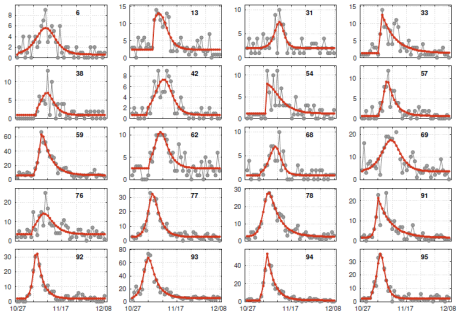


Events: France

Grey points: data (S. Roché);

Red curves: model (L. Bonnasse-Gahot *et al*, in preparation)

Fit: only 7 parameters for reproducing the timecourse city per city for all France.



Events: *Départements* with the largest rioting activities

Complex systems in urban studies

Social segregation

- ▶ simple models: Schelling model and variants
- ▶ housing market with social influences

T. C. Schelling

Thomas Crombie Schelling (1921 -.)



economist & foreign policy adviser

Distinguished University Professor at the University of Maryland, in the Department of Economics and the School of Public Policy

Nobel prize in Economic science 2005 (The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel)

T. C. Schelling

From Schelling biography on the Nobel prizes web site, about his contribution to the understanding of negotiation, **cooperation and conflicts**, through original extensions of game theory:

"I was trying to get game theorists to pay more attention to strategic activities, things like promises and threats, tacit bargaining, the role of communication, tactics of coordination, the design of enforceable contracts and rules, the use of agents, and all the tactics by which individuals or firms or governments committed themselves credibly.

I don't think I had any noticeable influence on game theorists, but I did reach sociologists, political scientists, and some economists."

T. C. Schelling

Schelling models and beyond

- ▶ Segregation
 - ▶ Schelling segregation (ethnic segregation)
 - ▶ A single neighborhood: agents with heterogenous preferences
 - ▶ Socio-spatial segregation: an agent based model
 - ▶ Phase diagram of the Schelling model
 - ▶ Schelling segregation in an open city
 - ▶ Adding economic constraints (income segregation)

Schelling's segregation model

References

T. C. Schelling, "A self-forming neighborhood model"

"Models of Segregation",
The American Economic Review, Vol. 59 (2), 1969

"Dynamic Models of Segregation",
Journal of Math. Sociology 1:143-186, 1971

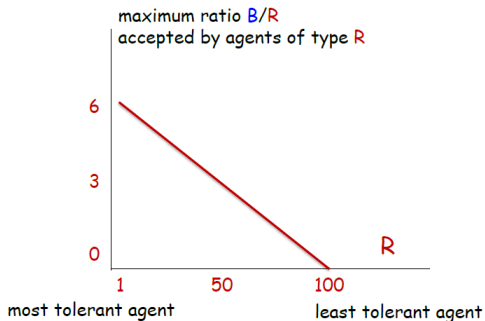
From micromotives to macrobehavior (Norton & Cy, 1978)

Schelling's segregation models (1971)

A single neighborhood

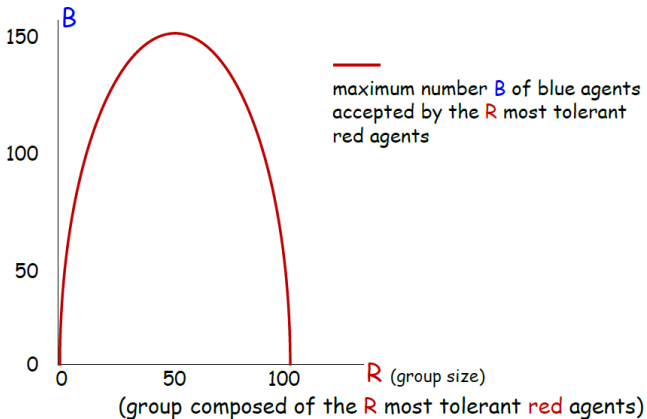
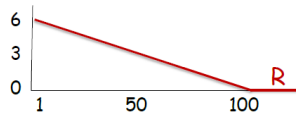
Issue: to join or not a club. Two types of agents, **Red** and **Blue**.
Every agent accepts (or desires) to join this neighborhood (club) provided the fraction of agents of the other color is at most equal to his idiosyncratic tolerance threshold.

Heterogeneous preferences



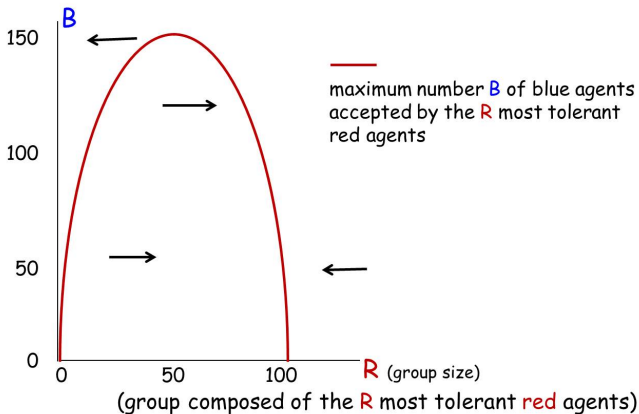
Schelling's segregation models (1971)

A single neighborhood



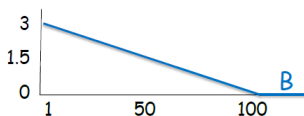
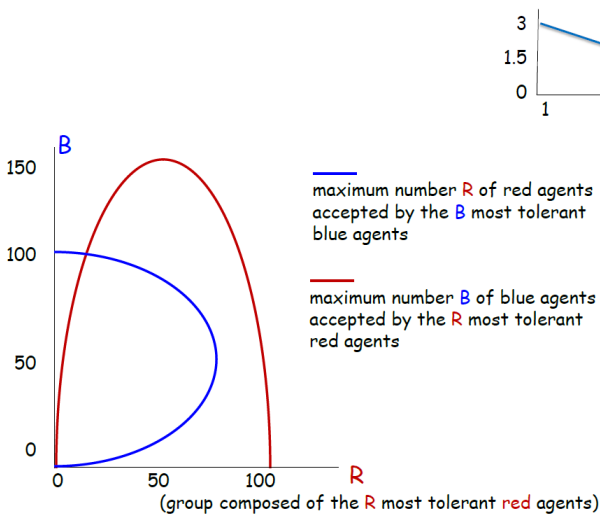
Schelling's segregation models (1971)

A single neighborhood



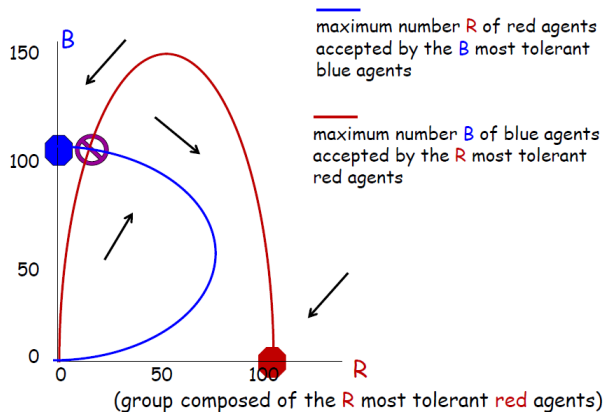
Schelling's segregation models (1971)

A single neighborhood



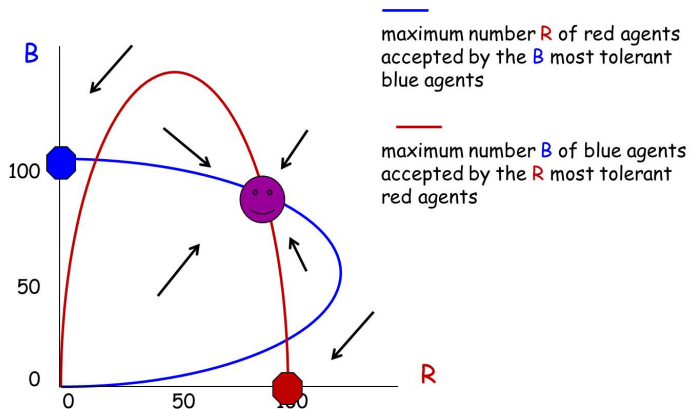
Schelling's segregation models (1971)

A single neighborhood



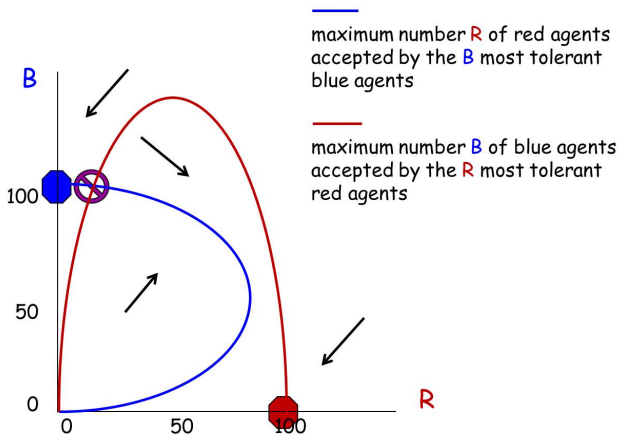
Schelling's segregation models (1971)

A single neighborhood



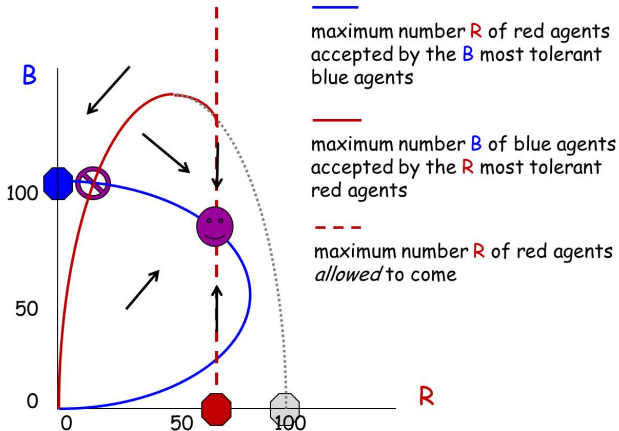
Schelling's segregation models (1971)

A single neighborhood



Schelling's segregation models (1971)

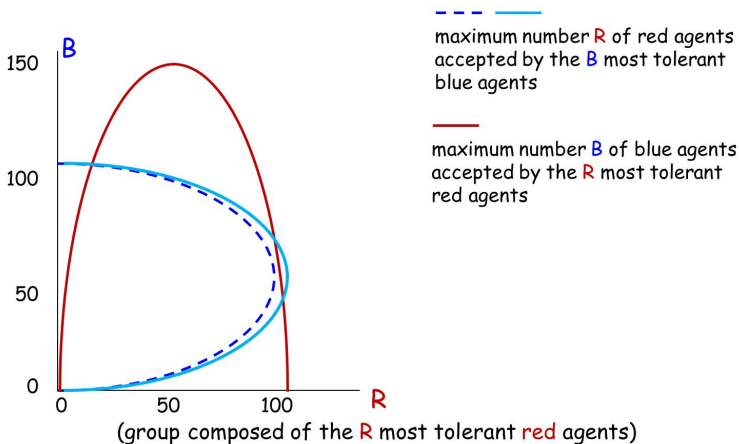
A single neighborhood



Schelling's segregation models (1971)

A single neighborhood

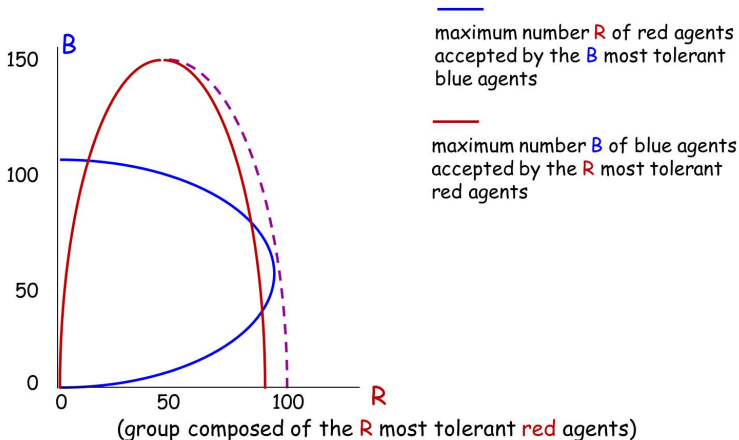
"Phase transition". Critical case: two examples of (blue) agents preferences, very close from one another. But (given the red preferences), one with, and one without, a mixed fixed point.



Schelling's segregation models (1971)

A single neighborhood

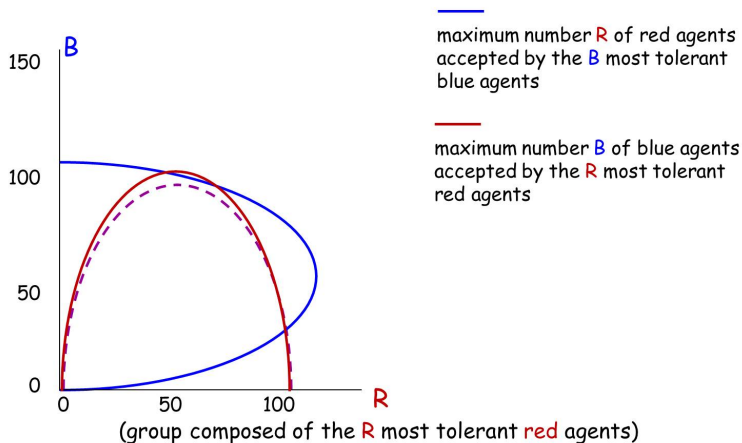
"Phase transition". Critical case: two examples of (red) agents preferences, very close from one another But (given the blue preferences), one with, and one without, a mixed fixed point.



Schelling's segregation models (1971)




A single neighborhood

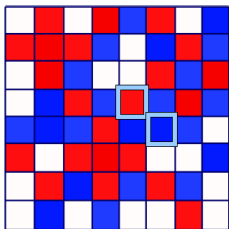
Symmetric example, exchanging the role of the red and blue agents: Critical case: two examples of (red) agents preferences, very close from one another, but (given the blue preferences), one with and one without a mixed fixed point.



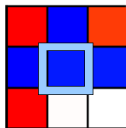
Schelling's socio-spatial segregation model

An agent-based model

- Initial random configuration of 2 types of agents:  &  and vacancies: 
- Satisfactory condition: $N_s > 1/3 (N_s + N_d)$
- Dynamics: unsatisfied agents move to the closest satisfying vacancy

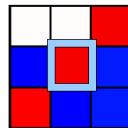


Moore neighbourhood: 8 sites



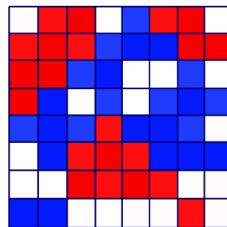
- 3 identical
- 3 different
- 2 vacancies

↓
satisfied



- 2 identical
- 4 different
- 2 vacancies

↓
unsatisfied



Schelling & agent based simulations

“Some vivid dynamics can be generated by any reader with a half-hour to spare, a roll of pennies and a roll of dimes, a tabletop, a large sheet of paper, a spirit of scientific inquiry, or, lacking that spirit, a fondness for games.”

T. C. Schelling, “A self-forming neighborhood model”, 1971

T. C. Schelling, *From micromotives to macrobehavior* (Norton & Cy, 1978)

Simulations

- ▶ Fixed equal number of red agents and blue agents, fixed number of vacancies
- ▶ Free boundary conditions
- ▶ Two parameters: tolerance T and density of vacancies ρ
An agent is satisfied iff $N_d \leq T(N_s + N_d)$
- ▶ Initial random configuration (two types of agents fully mixed)

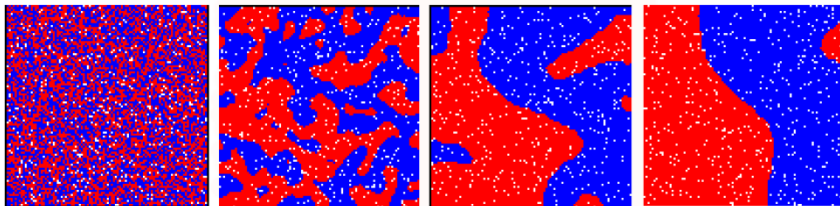


Figure: snapshots of the 'city' at different instants of time.

Here $T = 0.5$ and $\rho = 5\%$.

Studies of Schelling model

notably with tools from Physics

- ▶ Agents as interacting particles
 - ▶ Utility = internal energy \rightarrow force between agents
 - ▶ This force shapes the clusters - surface tension effects
D. Vinkovic & A Kirman, PNAS 2006
- ▶ Analogy with Ising spins
 - ▶ up/down spins = red/blue agents
 - ▶ minimization of energy - collective phenomena
Dall'Asta L, Castellano C, Marsili M (2008); Stauffer & Solomon (2007),...
- ▶ Variants of Schelling's model
 - ▶ Other preferences
 - ▶ stochastic decision rule ("*temperature*")
Pancs & Vriend 2007, Grauwin et al 2009,...

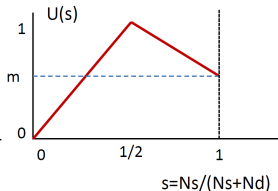
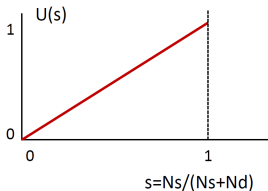
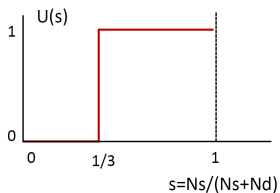
Schelling model: Variants

Utility of agents

Left: Schelling original model (tolerance threshold=1/3)

Middle and Right: Utility depending continuously on the fraction of similar neighbours

Right: Agents with a strict preference for a mixed neighborhood



Critical value of m :

$m < m_c$: no segregation,

$m > m_c$: segregation as in the original Schelling model

Pancs & Vriendt, 2007; Goffette-Nagot et al, 2009

Phase diagram of the Schelling's model

Model

Square lattice $L \times L$ ($L \gg 1$),
Moore neighborhood (8 nearest neighbours)

- ▶ An agent is satisfied iff:

$$N_d \leq T(N_s + N_d)$$

where

N_d = number of dissimilar neighbours,

N_s = number of similar neighbours,

T = tolerance parameter

- ▶ Density: total number of agents = $(1 - \rho)L^2$
(ρ = density of lacunes)
- ▶ Moves: any randomly picked agent may move to any satisfying vacancy

Simulations

- ▶ Fixed equal number of red agents and blue agents, fixed number of vacancies
- ▶ Free boundary conditions
- ▶ Two parameters: tolerance T and density of vacancies ρ
- ▶ Initial random configuration (two types of agents fully mixed)

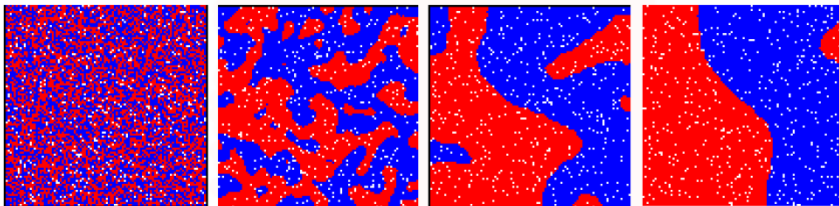
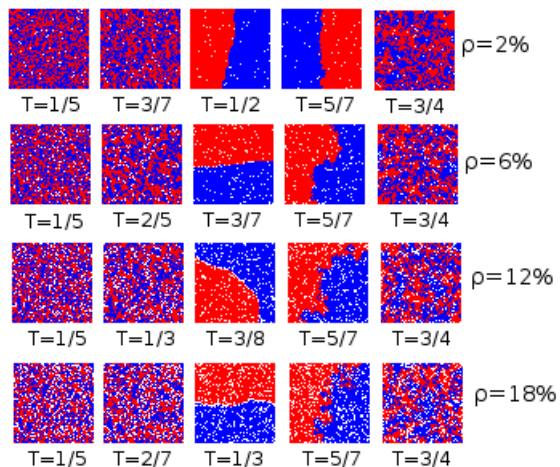


Figure: snapshots of the 'city' at different instants of time.
Here $T = 0.5$ and $\rho = 5\%$.

Simulations

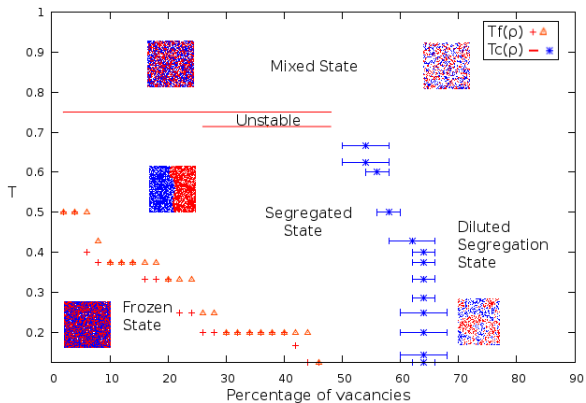
Stationary state for different values of the parameters T and ρ



L. Gauvin, J. Vannimenus and JPN, EPJB 2009

Analysis

Phase diagram



Control parameters: tolerance T & density of vacancies ρ

L. Gauvin, J. Vannimetus and JPN, EPJB 2009

Analysis

Segregated phase occupies a large domain of the phase diagram confirming Schelling's intuition concerning the genericity of the segregation phenomenon

- ▶ The abrupt mixed-segregated transition is reminiscent of the tipping point: rapid ethnic turn-over noticed by social scientists
- ▶ The state of diluted segregation could be relevant for low density suburban areas

Schelling's dynamics in an open city

- ▶ **Satisfactory condition:** an agent is satisfied if

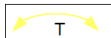
$$N_d - T(N_d + N_s) + D > 0$$

(N_d = number of dissimilar neighbours, N_s = number of similar neighbours)

- ▶ Two components:
 - **Neighborhood** - tolerance parameter, T
 - **Environment** - city attractiveness, D
- ▶ **Open system**
 - agents may leave or enter the city
 - \rightarrow fraction of empty sites not given, fixed on average by the control parameter D

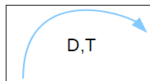
Schelling's dynamics in an open city

- Internal exchange: movement of an agent to a vacant site



$$[N_{df} - T(N_{df} + N_{sf})] - [N_d - T(N_d + N_s)] \leq 0$$

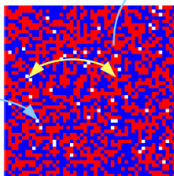
- External exchange: departure or arrival of an agent



$$N_d - T(N_d + N_s) + D \leq 0$$

Agents leaving the system

Reservoir with agents



Link with statistical physics models

For each site i , define the "spin-1" variable:

$$c_i = \begin{matrix} +1 & -1 & 0 \end{matrix}$$

and for the city the "energy" E by

$$E = \sum_i \left[- \left[\sum_{\langle j \rangle} c_i c_j + (2T - 1) \sum_{\langle j \rangle} c_i^2 c_j^2 \right] + D' c_i^2 \right]$$

Neighbour contribution

Environment contribution

$$D' = 2D$$

$$K = 2T - 1$$

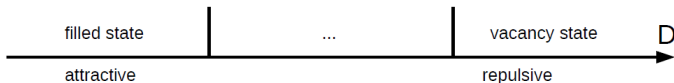
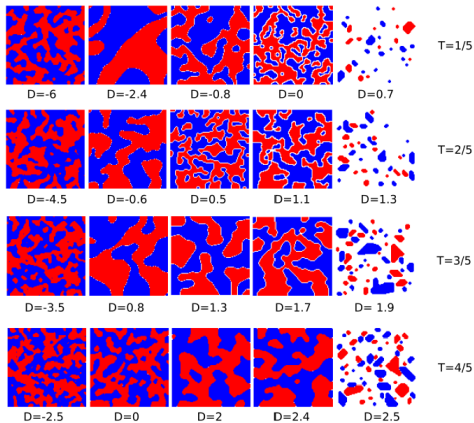
One can show that the dynamics prevents the energy from increasing during the moves.

The model defined by this energy is known in physics as the **Blume-Emery-Griffiths** (BEG) model, and the dynamics is then formally the one of the BEG model at zero temperature, with some kinetic restrictions.

Schelling's dynamics in an open city

Numerical simulations

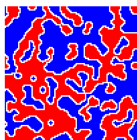
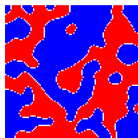
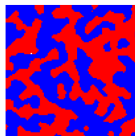
Stationary state for different values of the parameters T and D



Schelling's dynamics in an open city

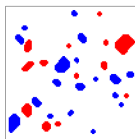
Stationary states

Negative D , attractive city: no empty site left. Segregation with direct contact between the two type of agents - the environment is so welcoming that agents prefer to be there even with dissimilar neighbours.



Intermediate values of D : lacunes appear and create borders between the two types of agents. Borders may have various shapes and widths.

Large D values: predominant vacancy state. The environment is strongly unwelcoming, only small and compact aggregates of similar agents remain.



Schelling's dynamics in an open city

Perspectives

Competition between the satisfaction of the agents regarding their social neighborhood and the environment.

- ▶ Strong links help avoiding massive “exodus”. Ex: Chicago (Wilson & Taub, “There goes the neighborhood”)
- ▶ Diluted state at high D : only a few groups of similar agents remain. Ex: New-Orleans
- ▶ There actually preexists structural borders (roads, parks,...). How would they influence these results?
Infrastructures may affect segregation: Paris ring road (R. Escallier, “Les frontières dans la ville”, Cahiers de la Méditerranée)

Ref: L. Gauvin, JPN and J. Vannimenus 2010

Housing market

Income segregation

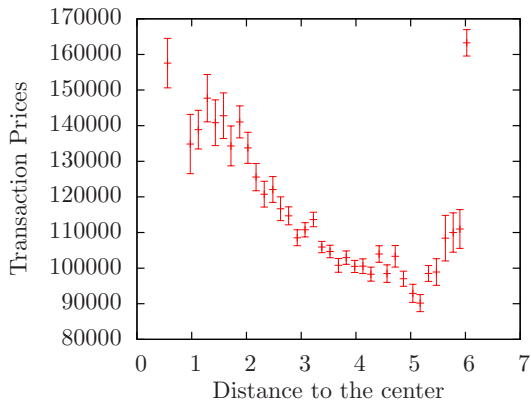
- Beyond Schelling's dynamics: more complex models
 - ▶ Portugali, Benenson, Omer, 1996: taking into account cognitive aspects (intentions vs actual behavior) with application to Israel cities
 - ▶ taking into account economic constraints: Kirman; Gauvin, Vignes, JPN
- In the following: **income** segregation & housing market

Housing market

Paris data

As a first approximation: prices decrease from the center.

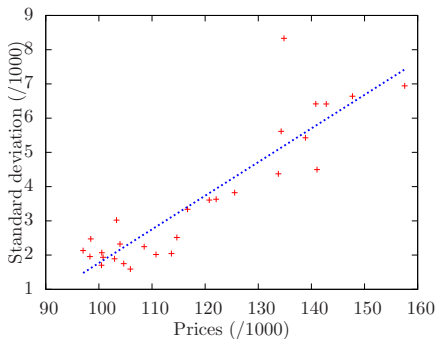
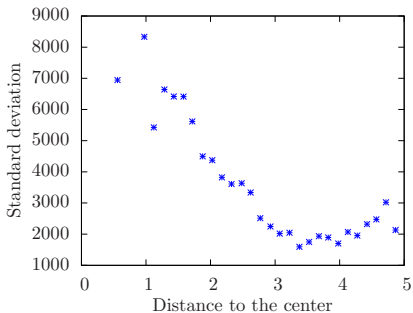
“hotspot”: 16th arrondissement.



Housing market

Paris data

variance increases with the mean price



Housing market

Model

Don't buy the house, buy the neighborhood (Russian proverb)

Agent-based model and partial differential equations

- Heterogeneous agents: K different **income levels** (→ social categories)

- **Attractiveness** of the locations $A_k(X, t)$ specific to each location X and function of the social category k ($k \in \{1, \dots, K\}$); allows to describe the interactions between space and agents.

Components:

- ▶ **Intrinsic** attractiveness $A^0(X)$ (quality of the apartment and of local amenities)
- ▶ **Social** component (externalities): depends on the social preferences of the agents
→ attractiveness evolves according to agents behaviour
- ▶ **Local diffusion**: a location becomes more attractive if the attractiveness of a nearby location increases.

- **Market dynamics**: "search" dynamics - agents want to buy where the attractiveness is the largest.

Attractiveness dynamics:

$$\partial_t A_k(X, t) =$$

$$\omega(A^0(X) - A_k(X, t))$$

+

(term function of social preferences)

+

$$D \Delta A_k(X, t)$$

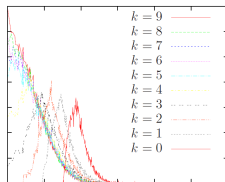
Housing market

Main results

Case studied in details (simulations + theory):
no diffusion, non saturated market;
social preferences: agents prefer neighbours with similar or higher income.

- **Emergence of social segregation** only if the weight of the social preferences is larger than some **threshold**.
- In that case, emergence of an income threshold.
- Only the 'riches' (those with income larger than the threshold), can buy everywhere, and concentrate in the best locations.
- However, **some social mixity remains** everywhere.
- Segregation threshold: function of the ratio $\min(\text{reserve price of buyers}) / \min(\text{offer prices from sellers})$
→ a pro-mixity policy must act on both the offer and the demand.

Case of an intrinsic attractiveness decreasing from the center:



Density of agents for each category
($k = 0, \dots, 9$), vs. Distance to the
center

Housing market

Main results

Confrontation with data: version of the model calibrated on the Paris housing market. Preliminary results.

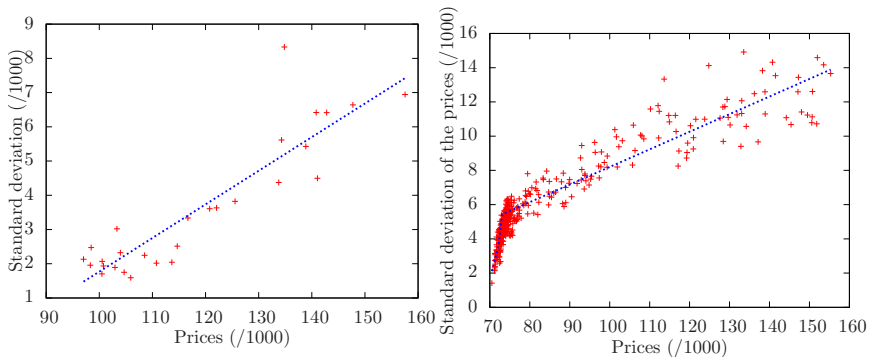


Figure: Standard deviation vs. mean price.

(left: data: Paris housing market

right: model)

Housing market

Perspectives

We have proposed a model going beyond the simple Schelling's framework, with a socio-economic approach.

⇒

General framework which allows for many variants;
Amenable to analytical analysis (at least in some regimes);
Allows for confrontation with empirical data.

Ongoing works - Perspectives:

- ▶ More realistic market mechanism
- ▶ More specific modelling of Paris and its area.
- ▶ Exploring other social preferences
- ▶ Emergence of hot spots: reaction-diffusion mechanism – diffusion of attractiveness to nearby sites

References

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- ▶ L. Gauvin, JPN and J. Vannimenus, "Schelling segregation in an open city: a kinetically constrained Blume-Emery-Griffiths spin-1 system", Phys. Rev. E 81, 066120 (2010)
- ▶ L. Gauvin, A. Vignes and JPN, "Modeling urban housing market dynamics: can the socio-spatial segregation preserve some social diversity", Journal of Economic Dynamics & Control (JEDC), 37, pp. 1300-1321, 2013

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<http://www.lps.ens.fr/~nadal/>

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Segregation



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Thank you