Urban social dynamics: Collective patterns of uncivil behavior

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ENPC – April 2018

From Statistical Physics to Complex Systems

Methodology

- Identification of typical properties (stylized facts - e.g. scaling laws)
- "Phase diagrams": in the space of parameters, determination of the boundaries between domains with qualitatively different behaviours.
  Boundaries: correspond to phase transitions, bifurcations
- Analysis of typical and optimal behaviours/performances
- Confrontation with empirical data,
  qualitatively: reproducing stylized facts
  quantitatively: data driven modelling
- Tools and concepts from Statistical Physics, Dynamical systems, Game Theory, Data science...
  mathematical modelling (ordinary and partial differential equations; discrete and continuous probabilistic models; game theoric models;...);
  numerical simulations ("agent based models").

Collective patterns of uncivil behavior

- hot spots of crime activities, eg burglaries
- gang patterns
- riots

Data driven modelling, partial differential equations and/or agent based models

Crime patterns
Predictive policing

- UCL, London, T Davies and S. Johnson:
  - road structure vs. burglary risk

- UCLA: cooperation between mathematicians (A. Bertozzi, M. Short), anthropologists (J. Brantingham), criminologists (G E Tita), and the police of Los Angeles (LAPD) and of other cities
  - hot spots, gangs, predictive policing

http://science360.gov/obj/video/f73df48e-c727-4771-a83b-91c05e8aaf01/science-behind-news-predictive-policing ("Science Behind the News: Predictive Policing", Anne Thompson, correspondent NBC Learn. NBCUniversal Media. 22 Feb. 2013)

Crime patterns

Predictive policing

mai 2015

LA VIGIE : LE MEILLEUR DU WEB

À LIRE SUR 20MINUTES.FR

21/05/2015 à 10h39

La gendarmerie a un nouveau logiciel pour prédire les délits

Signalé par
Camille Polloni

« Empêcher que les faits ne se réalisent », c'est l'ambition d'un nouveau logiciel prédictif expérimenté par la gendarmerie nationale pour anticiper les grandes tendances de la délinquance sur le territoire. Déjà testé en Bavière ou en Suisse, et utilisé en Californie, ce type de logiciel était encore inédit en France.

L'idée est d'analyser certaines catégories de délits fréquents – les cambriolages, les vols, les trafics de stupéfiants ou encore les agressions sexuelles – s'étant produits les cinq dernières années, pour tenter d'en tirer des régularités et de prévoir où et quand ils pourraient se renouveler dans les prochains mois.

Ce « lissage exponentiel » est traité par les chefs de service. « A eux ensuite d'adapter leurs moyens et d'exploiter au mieux ces renseignements criminels dans leurs zones », écrit 20 Minutes. Par exemple en augmentant le nombre de patrouilles aux abords des commerces.

Lire sur 20minutes.fr

Crime patterns
From social theory to modeling

Lawrence Cohen and Marcus Felson,
Social Change and Crime Rate Trends: A Routine Activity Approach
American Sociological Review, 1979

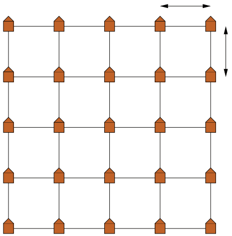
modelling:
agents behaviour $\leftrightarrow$ spatial field

Crime patterns
Burglaries

[M.B. Short, M.R. D'Orsogna, V.B. Pasour, G.E. Tita, P.J. Brantingham, A.L. Bertozzi, L.B. Chayes, A statistical model of criminal behavior, M3AS 2008]

Each house is described by its lattice site $s = (i, j)$ and a quantity $A_s(t)$ (attractiveness).

$$A_s(t) = A_s^0 + B_s(t) > 0$$



Probability a burglar commits a burglary:

$$p_s(t) = 1 - e^{-A_s(t)\delta t}$$

During each time interval δt, burglars perform exactly one of the following two tasks:

1. Burgle the home at which they are currently located, or
2. move to one of the adjacent homes (biased towards high $A_s(t)$).

When a house is burgled:

- The corresponding burglar is removed from the lattice.
- B_s is increased by a quantity θ, then decays over time.

Near-repeat victimisation: $B_s(t)$ spreads to its neighbours. Wandering burglers:

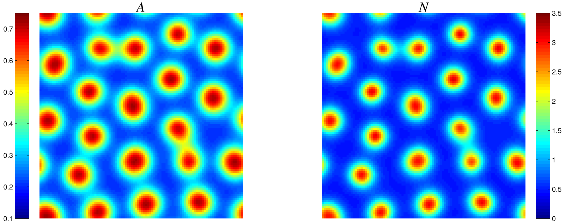
- Burglars come from sites they did not burgle in the previous time step
- New burglars are generated at each site at a rate Γ

From the agent based model, continuous time and space limits $\rightarrow$ Partial Differential Equations (PDEs)

$\Longleftrightarrow$ Reaction-diffusion models (appear in many cases of spontaneous pattern formation in physics and biology)

What do the solutions look like?

After 100 (nondimensional) time units
(*left: attractiveness* *right: density of burglars*)



periodic boundary conditions & initial conditions slightly perturbed from the uniform equilibrium state [A.B. Pitcher (2010)]

Riot contagion

Different types of riots

- Ethnic riots (LA, USA: Black vs Latinos, 1992; Bradford: south Asians vs Whites, 2001)
- Riots against the police (Brasil, 2006)
- Food riots (Egypt, 1977; Argentina, 1989, 2001)
- Ideological riots (Arab Spring)
- Mix types (e.g. French revolution: initially food riot: Women's march on Versailles)

- riots vs revolutions?
- spontaneous vs planned?

Riot contagion

Different types of riots

2005 French riots

- Wave of riots; more than 800 municipalities hit by the riots
- 3 weeks, starting Oct 27
- Casualties: 1 death, more than 200 wounded people
- 160 MEuros (insurances data)

2011 UK riots

- More than 2000 commercial premises hit by the riots
- Only 4 days
- Cost 300 M English pounds

The Guardian / London School of Economics: “Reading the riots“
`http://www.theguardian.com/uk/series/reading-the-riots`

The 2005 French riots

Les émeutes de 2005

video Christophe Rauzy / francetv info (3:39)

https://www.francetvinfo.fr/faits-divers/justice-proces/zyed-et-bouna/

video-emeutes-de-2005-les-trois-semaines-qui-ont-secoue-la-france_850519.html

The 2005 French riots

Almost 12 years ago: riots started on October 27, 2005

- wave of riots: 859 municipalities across all France hit by the riots
- triggering event: death of two youth trying to escape a police control
- riots started and spread in deprived neighborhoods ▸ more
- total duration $\sim$ 3 weeks
- mostly vehicles set on fire, but also burning of public buildings, confrontations between rioters and police, etc.
- Casualties: 1 death, more than 200 wounded people
- 160 MEuros (insurances data)

coverage by CNN:

(Remark: not a single city is correctly localized on this CNN map!)

A particular case of spontaneous social riots, as, e.g., food/hunger riots (UK 1766, 1801; Egypt 1977; Argentina 1989, 2001), ethnic riots (USA: Black vs Latinos, LA 1992...), 2011 UK riots...

Social riots

Specific features of spontaneous riots (stylized facts)

- spontaneous riots
  "C'était une révolte sociale et urbaine spontanée, pas du tout anticipée".
  It was a spontaneous social and urban revolt, not at all planned.
- social tension
  "Il y avait un ras-le-bol, des contrôles de police très fréquents, la ville était mise à l'écart... C'était une accumulation de tout ça, les révoltes."
  People were fed up with frequent police checks, . . . The riots, it was an accumulation of all that.
- triggering event ('shock')
  "La mort de Zyed et Bouna, deux ados qui pensaient qu'à aller à l'école et jouer au foot, ça a été la goutte d'eau"
  The death of Zyed and Bouna. . . , that has been the drop of water (that has made the vase overflow).

(quotations: testimony from actors of the 2005 French riots, as reported in the French press, AFP, Oct. 2015)

Social riots

Specific features of spontaneous riots (stylized facts)

- self-reinforcement mechanism, notably through imitation
  16 ans à l'époque, il est conscient aujourd'hui "d'avoir fait le mouton".
  16 years old at that time, he is conscious today to have acted as a sheep
- diffusion/propagation
  social networks, mass media
  propagation (or not) from one city to the other.
  2005 French riots: no displacement of rioters from city to city.
- ending the riot – short lived effect, but not clear why riots end
  • fear of the police? arrests? other deterrence factors?
  "Les médiateurs ont fait un gros boulot pour que ça se calme. Ils étaient là toutes les nuits."
  Mediators did a great job to calm people. They were there every night.
  • fatigue? cold weather?

Modeling riot contagion

- 'joining or not a riot': Schelling, Granovetter (70's)
  threshold models – Random Field Ising models
  M. B. Gordon, JPN, D. Phan and V. Semeshenko, M3AS 2009;
  J.-P. Bouchaud J Stat Phys 2013
- event history approach (econometric analysis)
  D. J. Myers 1997, 2000, 2010
- agent-based modelling: data driven modeling, 2011 UK riots
  P. Baudains, S. D. Johnson, A. M. Braithwaite 2013; T. Davies *et al* 2013
- reaction-diffusion approach, partial differential equations
  H. Berestycki, JPN and N. Rodriguez 2015
- epidemiological modeling
  S. L. Burbeck, W. J. Raine and M. J. A. Stark, 1978;
  this work:
  L. Bonnasse-Gahot, H. Berestycki, M.-A. Depuiset, M. B. Gordon, S. Roché, N. Rodriguez & JPN, 2018

Discrete choice under social influence
T. C. Schelling

Thomas Crombie Schelling (1921 -.)

economist & foreign policy adviser
Distinguished University Professor at the University of Maryland, in the Department of Economics and the School of Public Policy

Nobel prize in Economic science 2005 (The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel)

Discrete choice under social influence

Phase diagram

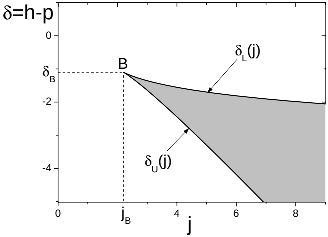
Thresholds models (Schelling, Granovetter)

$\leftrightarrow$ Random Utility Models (economics)
$\leftrightarrow$ Random Field Ising Model at zero temperature

Individual decision based on an idiosyncratic willingness to join, h_i
and on the fraction η of participants: 'YES' if $h_i + j\eta > p$

$h =$ mean willingness to... (attend the workshop / join the riot / ...)
$p =$ price, cost

$j =$ strength of the social influence



$h - p$ vs. j

Grey zone: domain of multiple equilibria

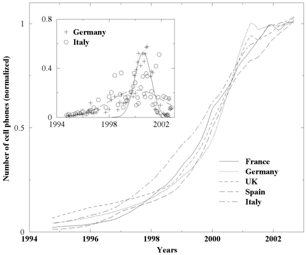
B: critical point

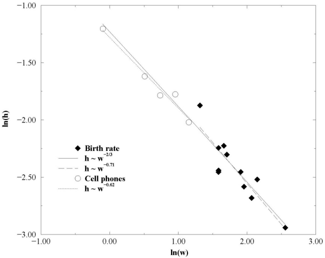
M. Gordon, JPN, D. Phan, V. Semeshenko 2009

Discrete choice under social influence

Vicinity of the critical point: scaling laws

max slope h vs width w of the transition: $h \sim w^{-2/3}$



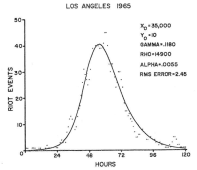


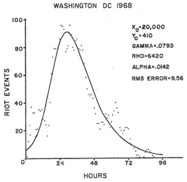
Michard & Bouchaud 2005

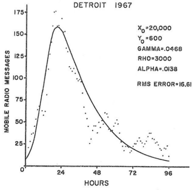
Modeling riot contagion
The epidemiological approach

- epidemiological modeling
  S L Burbeck, W J Raine and M J A Stark, 1978

Los Angeles (1965), Detroit (1967), and Washington, D.C., (1968) riots.







`` A new approach to the study of large-scale urban riots has resulted in the discovery of remarkably coherent patterns in the distribution of riot events over time. Patterns within three major riots suggest that the dynamics of the spread of riot behavior during a riot can be fruitfully compared to those operative in classical epidemics.''

``We therefore conceptualize riots as behavioral epidemics.''

The 2005 French riots: Data!

Source= daily police crime reports

from Oct 26, 2005 to Dec 08, 2005

44 police crime reports
typical size: 15 pages of text per report.

→ database: 6877 rows, 71 columns.
S. Roché, M.-A. Depuiset, M. B. Gordon, JPN, 2008-2010

→ for this work, dataset extracted from the database:

number of riot-like events, for each day, each municipality under police authority (essentially the most urbanized ones).

Total number of events: 6577.

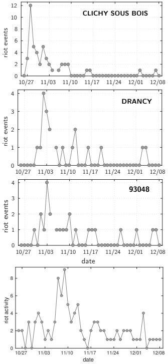
The 2005 French riots: Data!

Source= daily police crime reports.

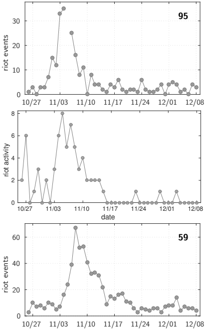
We look at the number of events per day, available for each municipality (municipalities under police authority - essentially the most urbanized ones.)

data from Oct 26, 2005 to Dec 08, 2005 (total number of events: 6577).

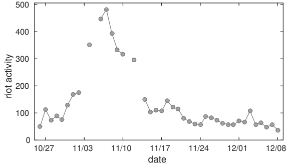
Communes



Départements



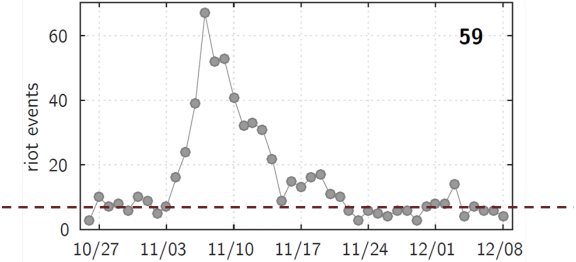
Country



“Commune” ≃ municipality

The 2005 French riots

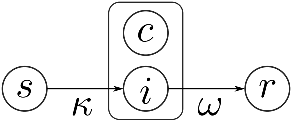
stationary background criminal activity

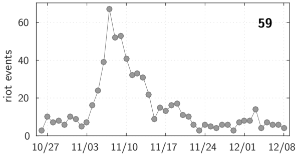


events taken into account in our dataset, for which one cannot say a priori if they are related to the riots.

typically, in France, 80 to 100 burnt cars every day, all year long.

Modeling: SIR model





no rebound in the data

$\rightarrow$ *no flux from recovered to susceptible*

- S: '**s**usceptible'
- I: '**i**nfected' (rioters)
- R: '**r**ecovered' (immune) (leaving the riot)
- C: background **c**riminal activity

- total number of events $= \lambda + \lambda_b$
- λ_b = background criminal activity (from population c)
- λ = (expected) number of events above this background noise (defining the rioting activity)
- Hypothesis:
  linear dependency between events and 'infected' population, $\lambda(t) = \alpha\, I(t)$

  $\rightarrow$ SIR model in term of λ
  (and of its conjugate variable, $\sigma \equiv \alpha S$).

Single site fits

As a first step, each site (each municipality, or each département, depending on the considered scale) is fitted separately
(as if there was a different triggering event specific to each site).

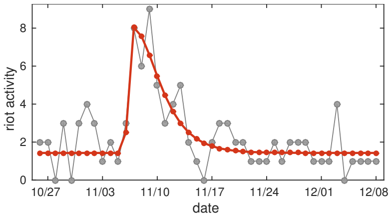
Hence, for each site:

- Background activity λ_b: given by the mean activity of the last two weeks (Nov. 25th–Dec 8th, 2005)
- Five free parameters: ω, $\sigma_0 = \alpha S_0$, β, t_0, A
  shock: at a time t_0, $I(t_0) > 0$, that is $\lambda(t_0) = A > 0$.
- Observations = discrete data. Assuming Poisson statistics, with mean λ, fit based on Maximum likelihood estimation.

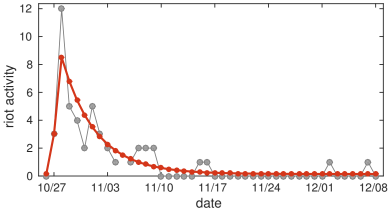
▸ Appendix

Single site fits – municipality examples

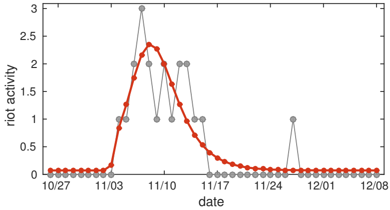
Marseille



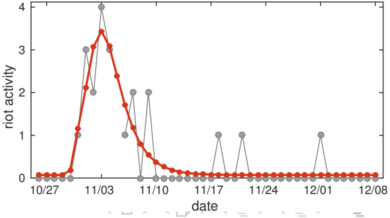
Clichy-sous-bois



Blois

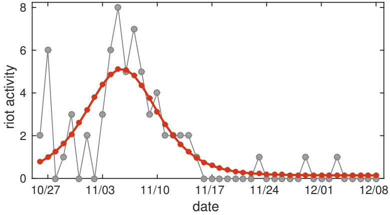


Cergy

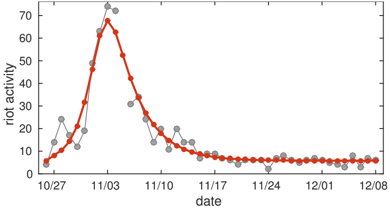


Single site fits – département examples

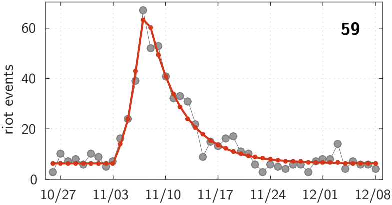
Eure (27)



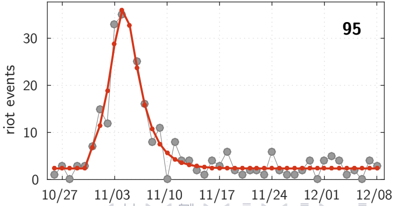
Seine-Saint-Denis (93)



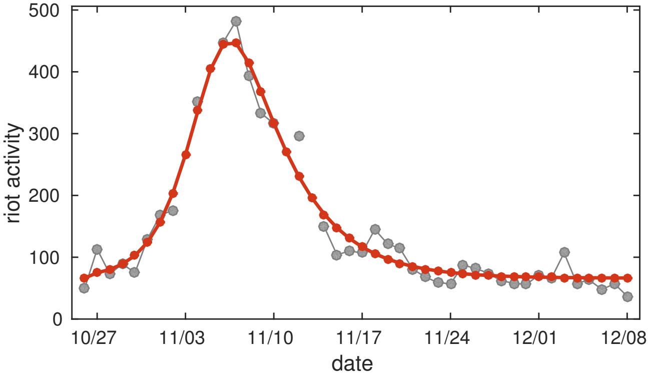
Nord (59)



Val-d'Oise (95)



Single site fit – whole country

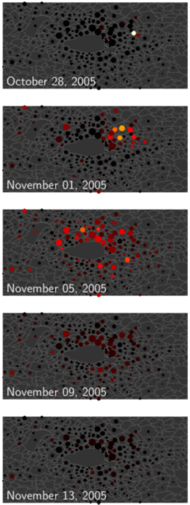


Single site fits = smoothing the data $\rightarrow$ visualization of the wave in the area around Paris

- for each site, the model allows to both fill in the blanks and smooth the data
  $\rightarrow$ makes diffusion phenomenon much clearer
  video *snapshots: one image every 4 days*

  https://static-content.springer.com/esm/art%3A10.1038%2Fs41598-017-18093-4/MediaObjects/41598_2017_18093_MOESM1_ESM.avi

- Major limitation: cannot account for the whole process; fitting all the municipalities amount to considering $859 \times 5 = 6013$ free parameters!



Global model

A single model with less than 10 free parameters

Spatial SIR type model (but no displacement of individuals).

Only one site, k_0 (Clichy-sous-Bois) receives a shock at a time t_0 (Oct. 27)

$t < t_0$, $\lambda_k(t) = 0$, $\sigma_k(t) = \sigma_{k0} > 0$; $\lambda_k(t_0) = A\,\delta_{k,k_0}$

$$\begin{cases} \dfrac{d\lambda_k}{dt} = -\omega\lambda_k(t) + \sigma_k(t)\,\Psi(\Lambda_k(t)) \\ \dfrac{d\sigma_k}{dt} = -\sigma_k(t)\,\Psi(\Lambda_k(t)) \\ \Lambda_k(t) = \sum_j w_{kj}\,\lambda_j(t) \end{cases}$$

Λ_k = global activity as seen from k

$\Psi(\Lambda)$ =

probability to join the riot given Λ

(non linear function: must saturate at

some value ≤ 1 for large Λ)

Ψ in the linear regime:

$$\begin{cases} \dfrac{d\lambda_k}{dt} = -\omega\,\lambda_k(t) + \beta\,\sigma_k(t)\,\Lambda_k(t) \\ \dfrac{d\sigma_k}{dt} = -\beta\,\sigma_k(t)\,\Lambda_k(t) \end{cases}$$

Geography matters:
$w_{kj} = W\left(d_{kj}/d_0\right)$
with d_{kj} = distance between k and j

e.g. $W(y) = (1+y)^{-\delta}$ or:

$W(y) = \mu + (1-\mu)\exp(-y)$

Global model

Initial conditions - Free parameters

- Initial susceptible population $\sigma_{k,0}$ for each site: we assume

$$\sigma_{k,0} = \zeta_0 N_k$$

where N_k is the size of a reference population (a poverty index that scales with total population size)
Chosen reference population: males aged between 16 and 24 with no diploma while not attending school (source: INSEE).

- Date of the triggering event, t_0: 27 October 2005.

Finally, with homogeneous free parameters,
in the linear case, only 6 free parameters: ω, β, ζ_0, d_0, δ, and A;
we will also allow for specific β values at a small number of sites, adding as many parameters;
for the nonlinear choice, instead of β, up to 4 parameters,
hence at most 9 free parameters.

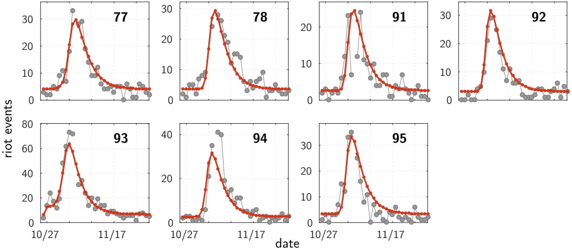
Global model: the wave in the area around Paris

Fitting the data at municipality scale

Working on Île de France: 1280 municipalities. Data available for the 462 municipalities under police authority – among those, 287 are mentioned in the database for at least one riot-like event.

Fit: dynamics for the 1280 municipalities (2×1280 coupled equations), parameters adaptation based on the 462 for which we have data.

Best fits with a power law decrease of the weights and a non linear function Ψ. We have here a total of 8 free parameters: $\omega, A, \zeta_0, d_0, \delta$, in addition to 3 for the non-linear function. Plots: results aggregated by Département.

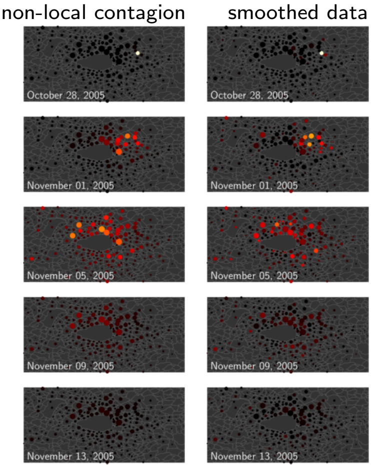


Global model: the wave in the area around Paris

Fitting the data at municipality scale

video

Snapshots: one image every 4 days $\sim 1/\omega$



`http://www.lps.ens.fr/~nadal/articles/riotwave/SI_Video_2.mp4`

reminder, smoothed data:

`http://www.lps.ens.fr/~nadal/articles/riotwave/SI_Video_1.mp4`

Fitting the data: the wave around Paris
Municipality scale

Reminder: Fit done with the dynamics generated for the 1280 municipalities (2×1280 coupled equations), parameters adaptation based on the 462 for which we have data.

What do we predict where we have no data? (municipalities under 'gendarmerie' authority, the smallest in term of population size and the least urbanized)

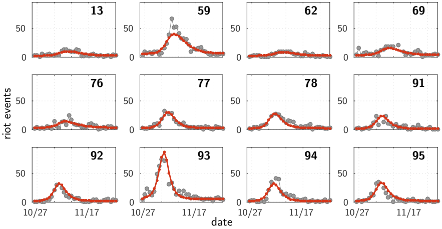
No rioting activity predicted in municipalities not present in the data base (λ remains < 1), except for one, Fleury-Merogis.

Looking back in the media: there has been at least one event!

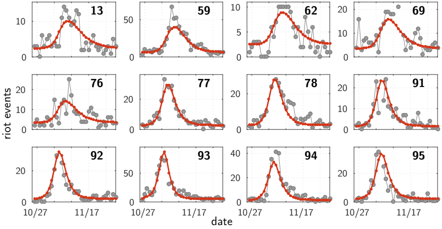
Global model: the wave accross the whole country

Fit at department scale. Plots: major départements

(absolute scale)



(same plots, relative scales)



All départements: ▸ here

Computational pb: more than 36000 municipalities!

Hence,

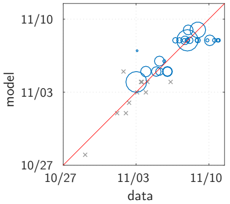
fit at département scale

2×93 coupled equations

Here 9 free parameters:

linear case, but with specific values of the susceptibility β for 3 locations: départements 93, 62 and 13.

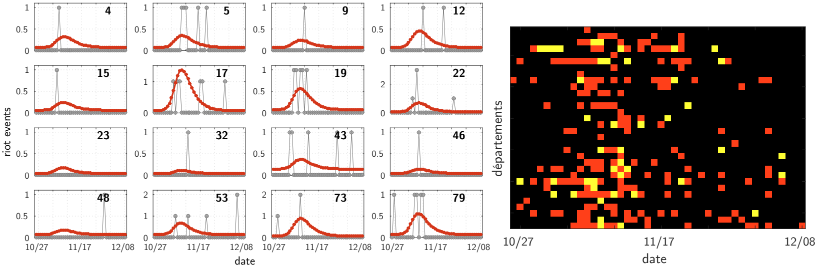
date of max of activity: model vs data



Global model: the wave accross the whole country

Fit at department scale. Plots: minor départements

Even where the number of events is very small, the wave is correctly predicted to hit with a very small amplitude at the correct date.



Statistical significance: for "very small" = max number of events on every day does not exceed a value as low as two. And better and better significance when increasing the maximum number of events defining the set of minor sites.

Discussion & Perspectives

- 'wave': precise meaning and visualization thanks to modeling
- geography matters: geographical proximity
  but also, long range interactions: socio-cultural proximity.
- Poisson statistics ▸ here
- confidence intervals and prediction - preliminary results
- mathematical analysis: link with spatially continuous SIR models ▸ here

- media: coverage of riots and media influence on riots – preliminary results
- ending the riot: effect of weather? (global decrease in riot activity parallels a global decrease in temperature) – preliminary results: no!
- role of police forces? arrests? (ω?)
- specific nature and intensity of the events? (e.g. burned vehicles, fights with the police)
- susceptibility or other parameters vs. socio-economic data

References - Support

References

- M. B. Gordon, JPN, D. Phan and V. Semeshenko, “Discrete Choices under Social Influence: Generic Properties”, Mathematical Models and Methods in Applied Sciences (M3AS), 2009
- L. Bonnasse-Gahot, H. Berestycki, M.-A. Depuiset, M. B. Gordon, S. Roché, N. Rodríguez and JPN, “Epidemiological modeling of the 2005 French riots: a spreading wave and the role of contagion”, Scientific Reports, 2018 (`http://rdcu.be/EiXe`)

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`http://www.lps.ens.fr/~nadal/`

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-  Henri Berestycki (EHESS - CAMS, Paris)
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Appendices

ZUS

- **France**
- RMI / 100 inh. $= 3.63$
- Unemployment$= 9.8$
- < 20 years $= 25$

- **Metropoliean Fr with ZUS**
- Unemploy$= 10.3$
- Crime/ 1000 $= 65.6$
- School (foreign)$= 4.2\%$

- **Seine-St-Denis (93)**
- RMI $= 7.1$
- Unemployment$= 13.9$
- < 20 years $= 29.2$

- **ZUS**
- Unemployment=20.7
- Unempl.(15-25)$=$ 36-40%
- Crime/ 1000 inh$= 68$
- School (foreign)$= 12.1$

The 2005 French riots: Data

Riots hit poor neighborhoods

In 2005, there is 36 570 municipalities in metropolitan France, among which 470 have (at least) one ZUS

ZUS: *Zone Urbaine Sensible* – Urban sensitive zone, deprived neighborhood.

859 municipalities hit by the riots, among which 382 have a ZUS:
81% of municipalities having a ZUS are hit,
45% of municipalities hit by the riots have a ZUS
– hence $P(hit|ZUS) = 0.81$ vs $P(hit|no\ ZUS) = 0.013$

Number of events: total 6577,
among which 4909, that is 75%, are in ZUS-municipalities.

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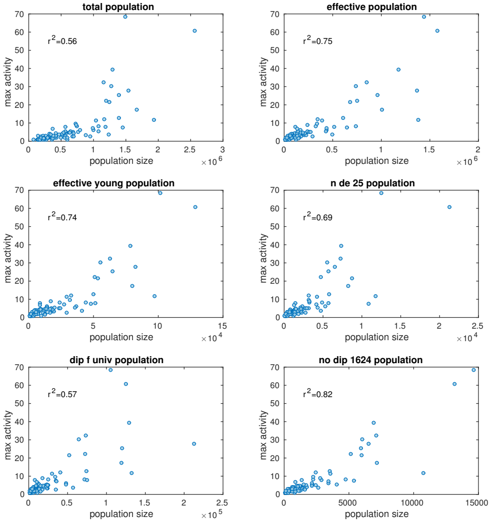
Population sizes

Maximum of rioting activity vs. the size of specific populations in the rioting municipalities, aggregated per departement:

total population; population of less than 25 year old citizen; population of women having a university diploma; population of males aged between 16 and 24 with no diploma while not attending school...

Best correlation is found for the population of males aged between 16 and 24 with no diploma while not attending school (reference population labelled `nodip_1624` in the following)



Population sizes

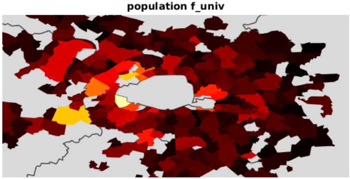
Paris area, qualitative comparison:

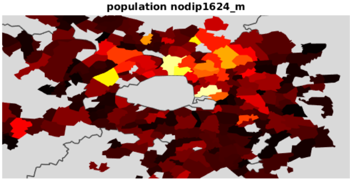
top, map of the number of women with university diploma

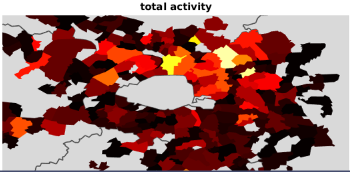
middle, map of the population of males aged between 16 and 24 with no diploma while not attending school

bottom: map of the total rioting activity

Paris city is the large grey domain at the center. The maps take into account all municipalities, around Paris, under police authority for which rioting data are available. The others (under 'gendarmerie' authority, mostly not strongly urbanized) are not shown (grey parts of the maps).

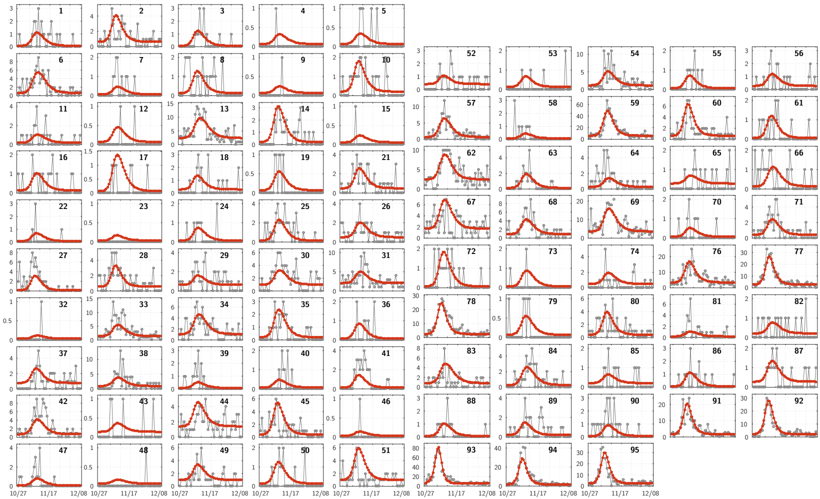






Global model : the wave accross the whole country

Fit at département scale – all départements



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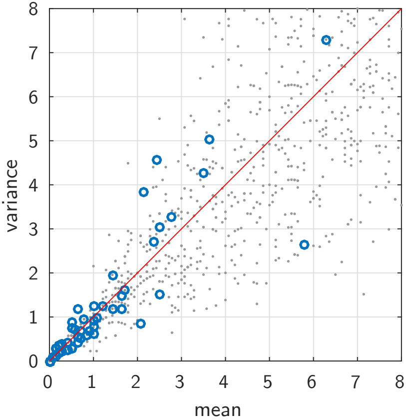
Poisson noise assumption

Poisson noise: looking at the flat tail (last two weeks)



(grey dots: surrogate data)

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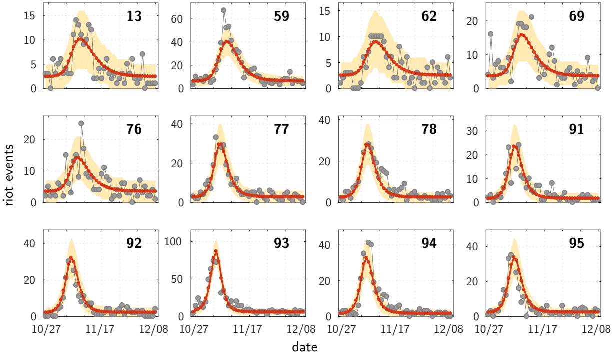
Highest density regions (HDR)



All of France, model calibration at the scale of the départements: data (dots) and model (continuous curves). Only the 12 most active départements are shown.

The light orange areas correspond to the 95% highest density regions.

HDR: If one draws a large number of realizations of a Poisson process with a given mean value λ, 95% of the points lie within the corresponding 95% HDR.

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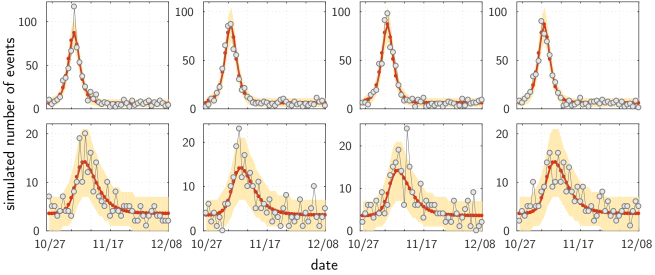
Surrogate data



Surrogate data: examples of Poisson samples (open circles) for two examples of rate curves (red curves). These two curves are taken from the model fit, top: for département 93, bottom: for département 76.
In each case, four different probabilistic realizations of Poisson noise are shown.

The light orange areas are the 95% highest density regions.

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Highest density regions (HDR)

We remind that we consider the observed data as probabilistic realizations of an underlying Poisson process, whose mean $\lambda(t)$ is the outcome of the model fit.

We have plotted the 95% Highest Density Regions (HDR, light orange areas) along with the means $\lambda(t)$ (red curves) of the Poisson processes. The rational is as follows. From fitting the model, for each site and for each date, we have a value of λ. If one draws a large number of realizations of a Poisson process with this mean value λ, one will find that 95% of the points lie within the corresponding 95% HDR.

For each value of the set of λs, outcome of the fit with the global, non local, model, we estimated the corresponding 95% HDR thanks to a Monte Carlo procedure.

Next, we look where the actual data points (grey points) lie with respect to the HDR. First, one sees that the empirical fluctuations are in agreement with the sizes of the HDRs. Second, remarkably, one finds that the percentage of data points outside the HDR is 9%, a value indeed close to the expected value $100 - 95 = 5\%$ (expected if both the fit is good and the noise is Poisson).

A closer look at the plots suggest a few large deviations, such as day 2 in département 69, that might correspond to true idiosyncrasies, cases which cannot be reproduced by the model.

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Links to the original spatially continuous SIR model

In the case of the linear approximation, our meta-population SIR model leads to a set of equations of a type similar to the space-continuous non local (distributed contact) SIR model, Kendall 1957 (non-local version of the Kermack-McKendrick SIR model):

$$\begin{cases} \dfrac{dI(x,t)}{dt} = -\omega\, I(x,t) + \beta\, S(x,t) \displaystyle\int K(x,y)\, I(y,t)\, dy \\ \dfrac{dS(x,t)}{dt} = -\beta\, S(x,t) \displaystyle\int K(x,y)\, I(y,t)\, dy \end{cases}$$

Known: dim. 1, homogeneous space – that is $K(x,y) = w(x-y)$,
travelling waves of any speed larger than or equal to some critical speed (Kendall 65).

Dim. 2, numerical simulations:
waves in the homogeneous case (Bailey 1967, Rodriguez-Meza 2012),

as well as in the heterogeneous one (Bonnasse-Gahot 2017).