Urban Population Densities

By Colin Clark

This branch of Geography does not yet seem to have received adequate quantitative study. The figures of density of population of cities sometimes published in reference books are practically meaningless. Arbitrary administrative decisions as to how much rural or semi-rural land is to be included within the city boundaries may completely alter the results. Fruitful study of urban population densities can only be made by drawing a map of the city and plotting, either by stippling or shading, the densities of population by the smallest component areas (Census Tracts or administrative sub-divisions) for which data are available. The task is laborious and the results lacking in precision ; but its application to a number of cities in different parts of the world and at different dates for which the evidence could be obtained has yielded some very interesting preliminary results.

The student of urban development will be greatly dependent upon Professor Griffith Taylor's *Urban Geography* (14) and the great learning therein contained. But this book is essentially designed to discuss qualitative criteria for ascertaining the various stages in the development of a town, with special emphasis upon the small town. Quantitative analysis of densities, with particular reference to great metropolitan areas, is hardly dealt with in it

This subject has attracted two brilliant statistical geographers—curiously enough, a long time ago, and there appears to have been little work since they wrote. The problem appealed to the versatile and painstaking genius of Mark Jefferson. In the *Bulletin of the Geographical Society* (predecessor of the *Geographical Review*) of 1909 (7), he published a series of density maps of the principal U.S. cities, based on the Census of 1900, and also of a number of European cities. By doing his work at this time he probably preserved much of the information from irreparable oblivion. For, while we still have, in the U.S. Census Reports, records of the population of each Ward and Assembly District in the big cities, it is very doubtful whether any information has been preserved which will enable us now to draw the boundaries of such districts.

Perhaps even more credit is due to the French scientist Meuriot, who published in 1898 *Des Agglomérations Urbaines dans l'Europe Contemporaine* (10), a book which remains to this day an outstanding study of nineteenth century urbanization in all its aspects. This book included a large number of density maps of contemporary European cities, and also traced their past development.

The preparation and detailed study of dozens of density maps is an overwhelming task unless we can begin with some hypothesis which will enable us to organize and simplify the data. We can begin with two generalizations the validity of which is now universally recognized:

1. In every large city, excluding the central business zone, which has few resident inhabitants, we have districts of dense population in the interior, with density falling off progressively as we proceed to the outer suburbs.

2. In most (but not all) cities, as time goes on, density tends to fall in the most populous inner suburbs, and to rise in the outer suburbs, and the whole city tends to " spread itself out."

The evidence assembled below appears to be sufficient to show that, in practically every case, the falling off of density, as we proceed to the outer suburbs, follows a simple mathematical equation of *exponential* decline.

Let x be distance in miles from the centre of the city.

Let y be the density of resident population in thousands per square mile.

Then (except in the central business zone)—

$$y = Ae^{-bx}.$$

That the falling off density is an exponential function, as in the above equation, appears to be true for all times and all places studied, from 1801 to the present day, and from Los Angeles to

Budapest. This maintenance of the exponential relationship is, however, compatible with very different rates of decline of density, as measured by the co-efficient b. A high value of b means that density will decline sharply with increasing distance from the centre, i.e., a compact city; a low value means that density declines more slowly, and the city is more spread out. It is clear that b is largely dependent upon the costs of intra-urban transport, or, more precisely, the cost of travelling in relation to the average citizen's income. Only where this cost is low can citizens afford to " spread out." The co-efficient A, on the other hand, measures the density or, shall we say, degree of over-crowding, which the citizens are prepared to tolerate at the centre of the city. At the centre of the city in the formula, x is equal to zero and y, therefore, becomes equal to A. It is a hypothetical rather than an actual figure, because in fact the centre of the city is occupied by the business zone with few or no resident inhabitants. Nevertheless it remains a useful figure; it shows the point to which densities are tending, if we measure the densities of the inner residential suburbs and continue extrapolating them inwards to reach the centre of the city.*

To use this formula it was found convenient to calculate total population, and hence average density, in a series of concentric rings about the centre of the city, generally drawn at each mile radius. Where these circles cut the boundaries of the Census Tracts or other administrative divisions by which the original data were classified, arbitrary apportionments of population had to be made. This was done in proportion to the area to the tract lying in each ring, after known open spaces (e.g., parks and mountains) had been excluded. Where the tracts are very small, as for U.S.A., 1940, Liverpool, Manchester and Dublin, this does not introduce any serious error. For the remainder of the British data, and for the Australian data, the tracts were larger, while for the rest of the data not only were the tracts large but the exact density was not given, only the " density class " being shown.

If we were to perform the whole investigation afresh, it would obviously be better to plot, for each tract, the recorded average density against its mean distance from the centre of the city, as this would eliminate the errors due to the apportionment process, and give a better picture of the scatter about the regression line.

The determination of the co-efficients A and b is now comparatively simple. A diagram is prepared in which the horizontal co-ordinate is a distance in miles from the centre of the city to the mid-point of the ring under examination, and the vertical co-ordinate is the *natural logarithm*

* The two co-efficients A and b must also be mathematically related to the total population of the city If density of population, in thousands per square mile, at distance x from the centre of the city, is given by the formula—

$$y = Ae^{-bx}$$

then the total population within a radius r is given by—

$$\int_0^r Ae^{-bx}(2\pi x)\,dx$$

$$= 2A\pi\int_0^r e^{-bx}x\,dx$$

$$= 2A\pi\Big[-\frac{e^{-bx}}{b^2}(1+bx)\Big]_0^r$$

$$= \frac{2A\pi}{b^2}\{1-e^{-br}(1+br)\}$$

when $r = \alpha$ (i.e., to obtain the total population of the city, in thousands) this becomes—

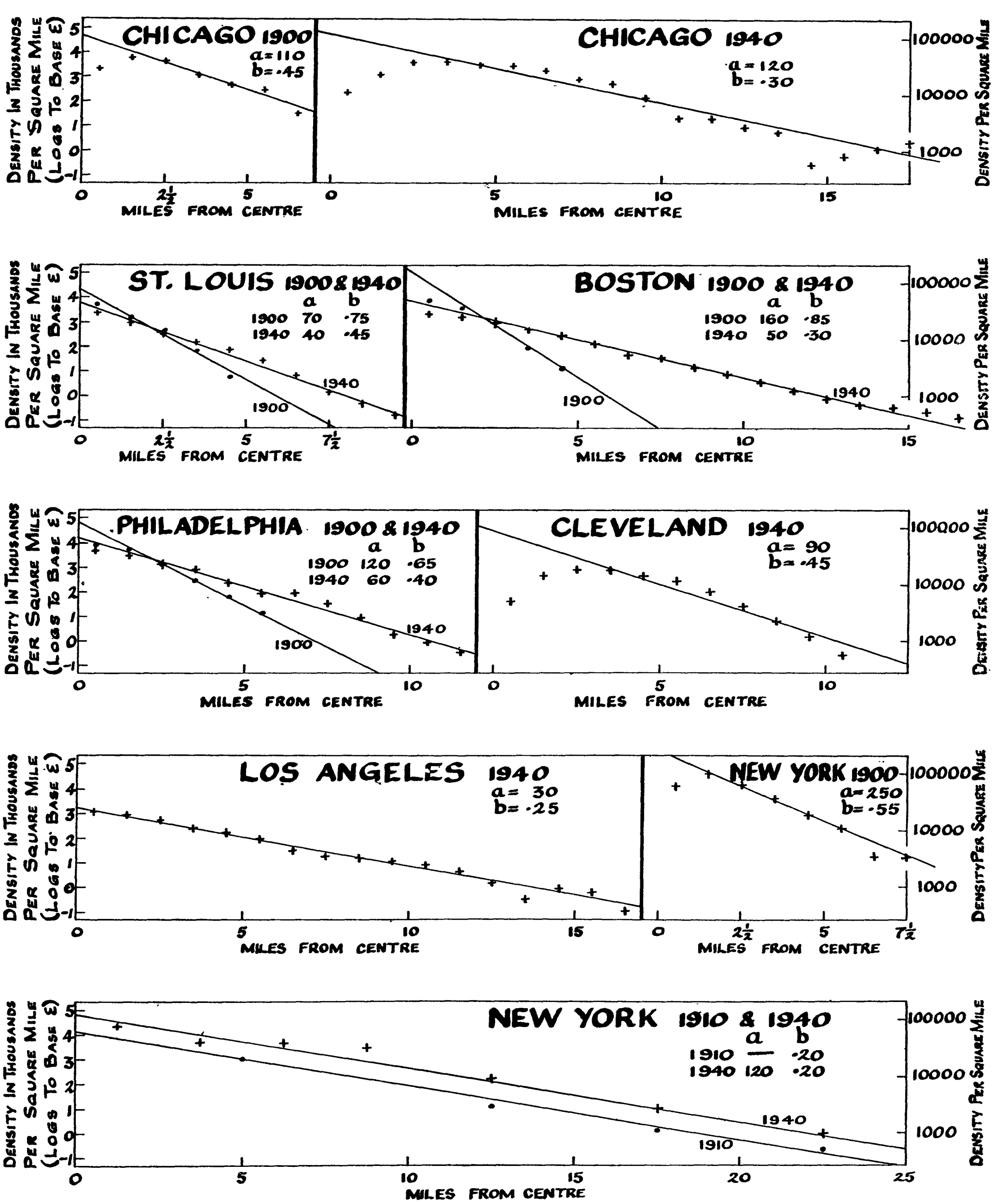
$$\frac{2A\pi}{b^2}.$$

This result, however, is still subject to two qualifications :

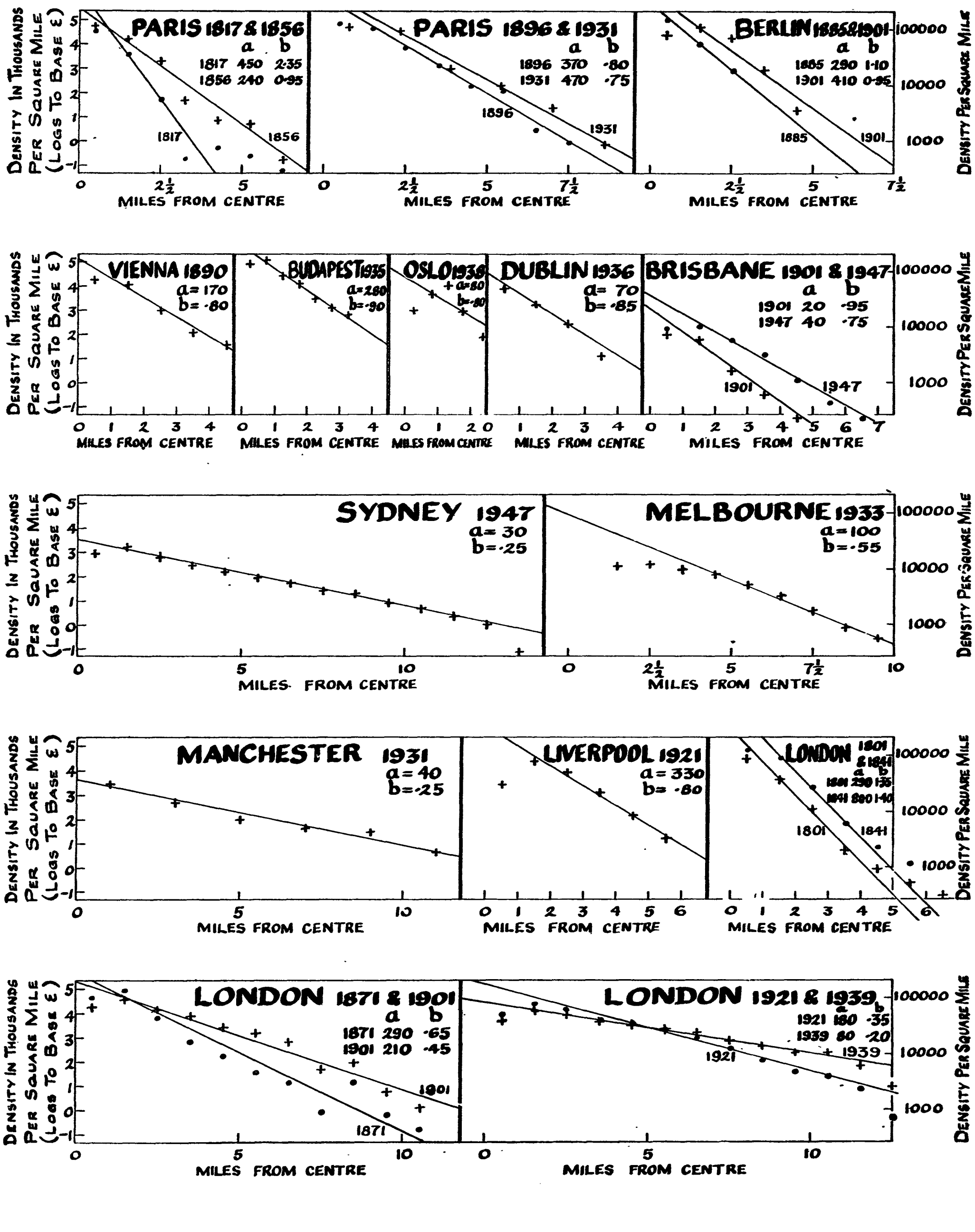
(i) We have made no allowance for the space in the central business zone, which is not available for residence.

(ii) We have assumed that the city can spread out uniformly and that all the land is available for residential building, i.e., no sea, estuaries, mountains, national parks, or the like.

Trying out the above formula allowing for these two considerations, would be a useful matter for further analysis. At this stage it can only be remarked that the values of A and b for various cities, ascertained below, do give at any rate the right order of magnitude for total populations.



DENSITY IN THOUSANDS PER SQUARE MILE (LOGS TO BASE e)
DENSITY PER SQUARE MILE
100000
10000
1000
CHICAGO 1900
a=110
b=·45
CHICAGO 1940
a=120
b=·30
MILES FROM CENTRE
ST. LOUIS 1900 & 1940
a b
1900 70 ·75
1940 40 ·45
BOSTON 1900 & 1940
a b
1900 160 ·85
1940 50 ·30
1900
1940
PHILADELPHIA 1900 & 1940
a b
1900 120 ·65
1940 60 ·40
CLEVELAND 1940
a= 90
b= ·45
LOS ANGELES 1940
a= 30
b= ·25
NEW YORK 1900
a=250
b=·55
NEW YORK 1910 & 1940
a b
1910 — ·20
1940 120 ·20
1910
1940



PARIS 1817 & 1856
a b
1817 450 2·35
1856 240 0·95
PARIS 1896 & 1931
a b
1896 370 ·80
1931 470 ·75
BERLIN 1885 & 1901
a b
1885 290 1·10
1901 410 0·95
VIENNA 1890
a = 170
b = ·80
BUDAPEST 1935
a = 260
b = ·90
OSLO 1938
a = 80
b = ·80
DUBLIN 1936
a = 70
b = ·85
BRISBANE 1901 & 1947
a b
1901 20 ·95
1947 40 ·75
SYDNEY 1947
a = 30
b = ·25
MELBOURNE 1933
a = 100
b = ·55
MANCHESTER 1931
a = 40
b = ·25
LIVERPOOL 1921
a = 330
b = ·80
LONDON 1801 & 1841
LONDON 1871 & 1901
a b
1871 290 ·65
1901 210 ·45
LONDON 1921 & 1939
a b
1921 180 ·35
1939 80 ·20
DENSITY IN THOUSANDS PER SQUARE MILE (LOGS TO BASE ε)
DENSITY PER SQUARE MILE
MILES FROM CENTRE

TABLE 1.

Parameters in the Expression $y = Ae^{-bx}$ *Relating Density of Resident Population in Thousands per Square Mile to Distance in Miles from the Centre of the City*

Region, City and Date	*A*	*b*
Australia—		
Brisbane		
1901	20	·95
1947	40	·75
Melbourne		
1933	100	·55
Sydney		
1947	30	·25
British Isles—		
Dublin		
1936	70	·85
Liverpool		
1921	330	·80
London		
1801	290	1·35
1841	800	1·40
1871	290	·65
1901	210	·45
1921	180	·35
1939	80	·20
Manchester		
1931	40	·25
Ceylon—Colombo		
1946	60	·40
Continental Europe—		
Berlin		
1885	290	1·10
1901	410	·95
Budapest		
1935	280	·90
Continental Europe (*continued*)—		
Oslo		
1938	80	·80
Paris		
1817	450	2·35
1856	240	·95
1896	370	·80
1931	470	·75
Vienna		
1890	170	·80
United States of America—		
Boston		
1900	160	·85
1940	50	·30
Chicago		
1900	110	·45
1940	120	·30
Cleveland		
1940	90	·45
Los Angeles		
1940	30	·25
New York		
1900	250	·55
1910	?	·20
1940	120	·20
Philadelphia		
1900	120	·65
1940	60	·40
St. Louis		
1900	70	·75
1940	40	·45

Sources of Information

U.S. Cities 1940—from Census Populations by tracts in the principal cities and by townships and other minor civil divisions in the remaining part of the metropolitan area.

Maps of census tracts and of civil divisions in the outer part of metropolitan areas, and their populations, given in the Census Reports on Populations, vol. i, Number of Inhabitants (15).

All U.S. figures for 1900 from Jefferson (7). New York figures for 1910 from Regional Planning Association; these do not analyse distribution within the first ten miles from the centre of the city.

Meuriot's data (10) used for Paris, Berlin and Vienna.

Figures for Paris for 1931 from Demangeon, *Paris—la Ville et sa Banlieue* (5).

Data for Budapest and Dublin from Density maps published in *Geographical Review* in 1943 (2) and 1946 (16) respectively, and for Liverpool from density map published in the *Journal of the Royal Statistical Society*, 1930 (8).

Figures for Manchester and all the surrounding metropolitan area from density data given in *City of Manchester Plan*, prepared by the City Council, 1945 (12).

Figures and map for Oslo from the Municipal *Statistisk Årbok* (13).

For London the Registrar-General of England and Wales publishes estimates by parliamentary divisions, used for 1939; and Census data for the same divisions for 1921. In the English Census of 1871 a special effort was made to preserve and render comparable figures of London's population by civil parishes back to 1801. The Parish boundaries were taken from maps published in Charles Booth's *Life and Labour in London* (3).

For Melbourne figures were taken from Dr. Fooks's book *X-ray the City!* (6).

For Sydney and Brisbane estimates were made direct from Census returns (1).

of the density, measured in thousands per square mile. In this way we can plot the equation as a straight line:

$$y = Ae^{-bx}$$
$$\therefore \log_e y = \log_e A - bx.$$

The calculated values of A and b are shown in Table 1 and diagrams are attached for all the cities for which information is available. It will be seen that in each case the data lie approximately along a straight line, except for the central business zone, where the resident population is always smaller than might have been expected. The value of A is determined from the point at which this line cuts the vertical axis, and the value of b from its slope.

These results do enable us to give a simplified classification of cities and of their trend of growth. If a metropolitan area is to have a high total population, it must either put up with a considerable degree of overcrowding in the inner suburbs, or it must spread itself out; this is the verbal expression of the mathematical equation given in the previous footnote. Spreading out is only possible where transport costs are low in relation to the citizens' income. A city with fairly high transport costs will have a fairly high value of b, and as its total population grows, the value of A (degree of overcrowding at the centre) must necessarily rise.

We naturally expect to find b at its maximum in those few records we have of the early nineteenth century, where it stood at 1·4 in London and perhaps as high as 2·35 in Paris. These were cities entirely dependent upon foot and horse transport, and even the latter was only for the few. There were horse buses in both cities, but they were not extensively used. The higher figure for Paris is probably determined by the longer hours of the Parisian worker, and his inability to walk the distance which some Londoners were willing to walk.

At the other end of the scale we get minimum values of b approaching 0·2 in present-day London, New York and Los Angeles, with Manchester, Sydney and Chicago not far behind. Their transport depends upon cheap underground railways in New York and London, surface electric railways in Chicago and Sydney, and largely upon private automobiles in Los Angeles.

But is is certainly surprising to see the similarity of the diagrams for Los Angeles and Manchester, two cities which one would believe, on first impressions, to be poles apart.* The co-efficient A is higher for Manchester (somewhat greater crowding at the centre) but their co-efficients b are very similar. Los Angeles claims to have achieved its dispersal through growing up in the automobile age and relying on that form of transport. Manchester and the adjacent towns grew up in the nineteenth and first years of the present century, and for transport relied upon steam trains and tramcars.† But they have managed to achieve almost the same degree of dispersal as Los Angeles. One reason is that factories and other places of employment are well distributed around Manchester. This is in violent contrast to Liverpool, where places of employment are largely centred along a narrow water-front, and where a large population has accumulated so impoverished and distressed by casual employment that it cannot afford the time or money for tram journeys.

Where we complete on one diagram figures for one city at two different dates, we can at once see the two possibilities for development, if the population is increasing. Either transport costs are reduced, enabling the city to spread out; or they cannot be reduced, in which case density has to increase at all points. In the former case we get a diagram of two intersecting lines, in the latter case two lines more or less parallel. An interesting example of the latter is the development in London between 1801 and 1841. There was no significant improvement in transport in this period, and density rose at all points. But when we go forward to 1871, with a widespread service of steam trains and horse buses, we notice a complete change of slope ; b has fallen from 1·41 to 0·66.

In Paris we can see this change as early as 1856. By 1896 the value of b for Paris had fallen to 0·79. But since then there has been little further fall. The Métro has been electrified, but the fares are high in relation to Parisian wages; more important is the belief of the Parisian worker

* After visiting each of these cities twice during recent years, one does however get the impression that in its production of smoke and industrial litter Los Angeles is now doing its best to imitate the physical appearance of Manchester, so far as its climate will allow.

† Until about twenty years ago, when some of the tramcars were replaced by buses, it was possible to go right across the County of Lancashire, from the sea to the Yorkshire border, in a succession of tram journeys. The present writer has performed this feat.

that his two-hour lunch interval should be devoted to its proper purpose, and not wasted in travelling; for him to have to live at such a distance from his work that he has to eat his lunch away from home is a real hardship.

In 1900 Chicago had the lowest value of b for all American cities. It is interesting to read Jefferson's comments on the high degree of dispersal, by the standards of that time, which Chicago had already achieved, and the good health (again by the standards of that time) which its inhabitants enjoyed. London in 1901 had a similar value of b ; but its value of A was nearly twice Chicago's, and severe overcrowding prevailed in the inner suburbs. Between 1900 and 1940 we can trace heavy declines in the value of b in each of the American cities examined. The particularly violent decline in Boston is perhaps attributable to the electric railway.

For New York it would be of great interest to obtain more detailed figures for 1910, if they are anywhere to be found. We have no data for the inner suburbs, but those for the outer suburbs show that already by this date a distribution with a very low value of b generally obtained. The Subway and electric services on the main line railways were in use by this date, and fares were low in relation to New York wages. But it is particularly interesting to note that between 1910 and 1940 little further reduction of transport costs has been possible, and as a result the city has had to grow by increasing density at nearly all points.

We may conclude with some isolated evidence of urban population densities at other times and places, and compare them with the figures we have here. The world's first urban development is believed to have been the city of Ur, and Sir Leonard Woolley (17) estimates that this city in its mature phase about 2,000 B.C.,* covered 4 square miles of compact building area, with a population of 500,000, or 125,000 to the square mile. Something of this sort seems to have been the maximum density which our remote ancestors would tolerate. Professor Demangeon, in his book, gives detailed measurement of the Parisian city boundaries and population in 1329, and the density works out at 142,000 per square mile. Professor Judges (9) has prepared a population map of all London parishes in 1695. The innermost residential area (within one-quarter of a mile of the London Stone) had a population density of 140,000 per square mile, but outside this one-quarter-mile radius it fell to approximately 120,000.

There seems to be statistical evidence for Lewis Mumford's attack (11) on the nineteenth century industrial city, which, he claimed, compelled people to live at densities unheard of in previous cities. In Paris by 1861, for instance, the population of the four innermost *arrondissements* had risen to 180,000 per square mile—which is also the population of the most densely populated areas in present-day Budapest. But a world record, which it is to be hoped will never again be challenged, was established in Jefferson's studies, which showed that some parts of the Lower East Side in New York in 1900 were carrying population at the rate of 350,000 per square mile.

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* It is believed to have been at about this date that Abraham exchanged his home in Ur for the austere life of a desert nomad.