Mécanique Physique des Matériaux

Contraintes résiduelles

École des Ponts

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Lecture objectives

- Definition and misconception
- Physical origin of residual stresses
- Different scales
- Numerical approaches
- Some energetic concepts
- Experiments
- Applications to additive manufacturing

Lecture outline

Lecture outline

1. Forming and fabrication processes
2. Continuum mechanics
3. Transformation induced plasticity
4. Application to additive manufacturing

Forming and fabrication processes

- Selected examples
- Requirements
- Residual stresses

Selected examples

Variety of processes

- Casting
- Machining
- Forging
- Rolling
- Friction Stir Welding
- Welding
- Additive manufacturing

Selected examples

Casting

Selected examples

Forge

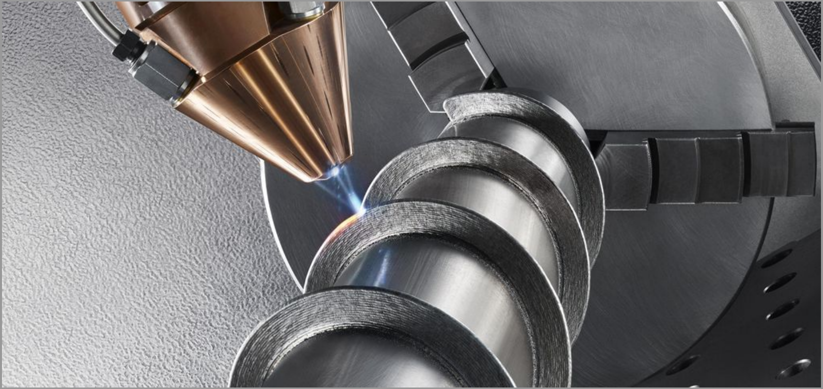
Selected examples
Rolling process

Selected examples

Welding

Selected examples

Additive manufacturing



Forming and fabrication processes

- Selected examples
- Requirements
- Residual stresses

Requirements

- Geometrical tolerances
- Defects
- Porosity
- Roughness
- …
- Phase transformations (past lecture)
- Residual stresses (this lecture)

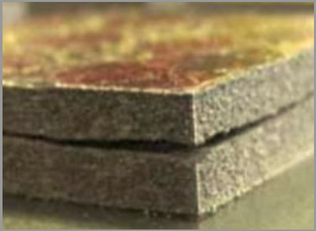
Forming and fabrication processes

- Selected examples
- Requirements
- Residual stresses

Residual stresses

A serious issue

Kempen et al. 2013



Residual stresses

Residual stresses

A serious issue (most of the time)

Sometimes on purpose

- Tempered glass
- Prestressed concrete
- ...

Predict and control

- Identify the physical origin : lower scale
- Develop a simplified model
- Coupling with thermo-mechanics
- Simulate the entire process

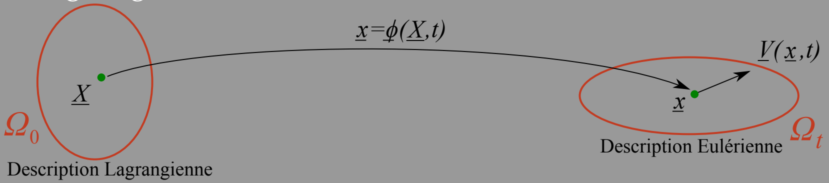
Lecture outline

Continuum mechanics

- Strain
- Stress
- Behavior
- Eigenstrain
- Residual stresses
- Navier equation

Strain

Lien Lagrangien / Eulérien



- $\underline{\underline{F}} = \underline{\underline{\nabla}}_{\underline{X}} \underline{\phi}$
- $\underline{dM} = \underline{\underline{F}}.\underline{dM}_0$
- $J = \det\left[\underline{\underline{F}}\right]$
- $d\Omega_t = J d\Omega_0$
- $\underline{\underline{C}} = {}^T\underline{\underline{F}}.\underline{\underline{F}}$

- $\underline{V} = \frac{\partial \underline{\phi}}{\partial t}$
- $\underline{\underline{\nabla}}_{\underline{x}} \underline{V} = \dot{\underline{\underline{F}}}.\underline{\underline{F}}^{-1}$
- $\text{div}_{\underline{x}} \underline{V} = \dot{J} J^{-1}$

- $\underline{V}$
- $\dot{\widehat{\underline{dM}}} = \underline{\underline{\nabla}}_{\underline{x}} \underline{V}.\underline{dM}$
- $\dot{\widehat{d\Omega_t}} = \text{div}_{\underline{x}} \underline{V}\, d\Omega_t$

$$\underline{\underline{e}} = \frac{1}{2}\left[\underline{\underline{C}} - \underline{\underline{1}}\right]$$

$$\underline{\underline{d}} = {}^T\underline{\underline{F}}^{-1}.\dot{\underline{\underline{e}}}.\underline{\underline{F}}^{-1}$$

$$\underline{\underline{d}} = \frac{1}{2}\left[\underline{\underline{\nabla}}_{\underline{x}} \underline{V} + {}^T\underline{\underline{\nabla}}_{\underline{x}} \underline{V}\right]$$

Continuum mechanics

- Strain
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Stress

Spaces

- Generalized velocity

$$\mathcal{C} = \{\underline{V} : (\underline{x}, t) \in \Omega_t \times \mathbb{R}_+ \mapsto \underline{V}(\underline{x}, t)\}$$

- Virtual velocity

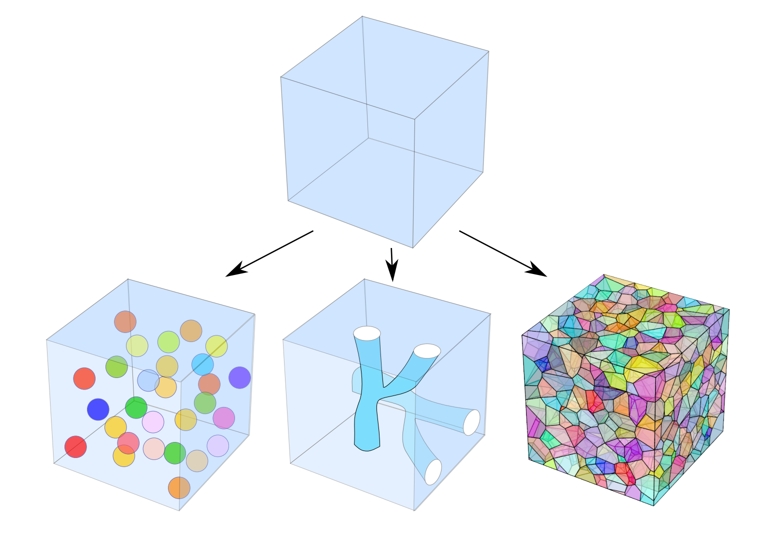
$$\mathcal{C}^* = \{\underline{V}^* : \underline{x} \in \Omega_t \mapsto \underline{V}^*(\underline{x})\}$$

- Rigid body motion

$$\mathcal{C}_R^* = \left\{\underline{V}_R^* : \underline{x} \in \Omega_t \mapsto \underline{V}_T + \underline{\underline{\omega}}.\underline{x},\ \forall \underline{V}_T \in \mathbb{R}^3 \ / \ \forall \underline{\underline{\omega}} \in \mathcal{M}_3^{as}\right\}$$

Stress

What does the material point represent ?



Stress

Cauchy

- Power of external forces

$$PVE(\underline{V}^*) = \int_{\Omega_t} \rho \underline{f}.\underline{V}^* \mathrm{d}\Omega + \int_{\partial\Omega_t} \underline{T}.\underline{V}^* \mathrm{d}S$$

- Power of internal forces

$$PVI(\underline{V}^*) = \int_{\Omega_t} \left(\underline{F}_0.\underline{V}^* - \underline{\underline{\sigma}} : \underline{\underline{\nabla}} [\underline{V}^*] \right) \mathrm{d}\Omega$$

- Power of acceleration forces

$$PVA(\underline{V}^*) = \int_{\Omega_t} \rho \underline{\gamma}.\underline{V}^* \mathrm{d}\Omega$$

- $\underline{\gamma}$ real acceleration field.

Stress

Conditions

- Consistence rigid body motion : no internal forces

$$\forall \underline{V}_T \in \mathbb{R}^3 \;\; \forall \underline{\underline{\omega}} \in \mathcal{M}_3^{as} \quad / \quad PVI(\underline{V}_T + \underline{\underline{\omega}}.\underline{x}) = 0$$

- Hence

$$\forall \underline{V}_T \in \mathbb{R}^3 \;\; \forall \underline{\underline{\omega}} \in \mathcal{M}_3^{as}$$

$$\int_{\Omega_t} \left(\underline{F}_0 . \left(\underline{V}_T + \underline{\underline{\omega}}.\underline{x} \right) - \underline{\underline{\sigma}} : \underline{\underline{\omega}} \right) \mathrm{d}\Omega = 0$$

- Hence

$$\underline{F}_0 = 0 \quad \text{et} \quad \int_{\Omega_t} \underline{\underline{\sigma}} : \underline{\underline{\omega}} \mathrm{d}\Omega = 0$$

- Anti-symmetric/symmetric tensors

$$\boxed{\underline{F}_0 = 0 \quad \text{et} \quad \underline{\underline{\sigma}} \in \mathcal{M}_3^{s}}$$

Stress

Postulate : principle of virtual power

- Stress $\underline{\underline{\sigma}}$ is symmetric, hence

$$\underline{\underline{d}}^*(\underline{V}^*) = \frac{1}{2}\left(\underline{\underline{\nabla}}\,[\underline{V}^*] + {}^t\underline{\underline{\nabla}}\,[\underline{V}^*]\right)$$

- Power of internal forces

$$PVI(\underline{V}^*) = -\int_{\Omega_t} \underline{\underline{\sigma}} : \underline{\underline{d}}^*(\underline{V}^*)\,\mathrm{d}\Omega$$

- Principle of virtual power

$$\boxed{\forall \underline{V}^* \in \mathcal{C}^* \quad PVI(\underline{V}^*) + PVE(\underline{V}^*) = PVA(\underline{V}^*)}$$

Continuum mechanics

- Strain
- Stress
- Behavior
- Eigenstrain
- Residual stresses
- Navier equation

Behavior

Energetic approach

- Balance equation for all possible evolutions

$$\boxed{\underline{\underline{\sigma}} : \underline{\underline{d}} - \rho \left(\dot{\Psi} + \dot{T} s \right) - \frac{\underline{q}.\underline{\nabla T}}{T} = D}$$

- Free energy $\rho\Psi$
- Example : isothermal elasticity

$$\underline{\underline{\sigma}} : \underline{\underline{d}} = \rho\dot{\Psi}$$

Behavior

Isothermal elasticity

- We have for all possible $\dot{\underline{\underline{e}}}$

$$\left(\underline{\underline{F}}^{-1}.\underline{\underline{\sigma}}.\left[\underline{\underline{F}}^{-1}\right]^T\right):\dot{\underline{\underline{e}}} = \rho \mathrm{sym}\left[\frac{\partial\Psi}{\underline{\underline{e}}}\right]:\dot{\underline{\underline{e}}}$$

- Hence

$$\boxed{\underline{\underline{\sigma}} = \rho\,\underline{\underline{F}}.\mathrm{sym}\left[\frac{\partial\Psi}{\underline{\underline{e}}}\right].\underline{\underline{F}}^T}$$

- Neo-Hookean : $\rho\Psi(\underline{\underline{e}}) = \frac{\mu}{2}\left(\bar{I}_1 - 3\right) + \frac{\lambda}{2}\left(J-1\right)^2$
- $J = \det(\underline{\underline{F}})$, $I_1 = \mathrm{tr}\left[\underline{\underline{C}}\right]$, $\bar{I}_1 = J^{-2/3}I_1$

$$\underline{\underline{\sigma}} = \frac{\mu}{J^{\frac{5}{3}}}\underline{\underline{F}}.\underline{\underline{F}}^T + \left(k(J-1) - \frac{\mu}{J^{\frac{5}{3}}}\frac{\mathrm{tr}\left(\underline{\underline{F}}.\underline{\underline{F}}^T\right)}{3}\right)\underline{\underline{1}}$$

Continuum mechanics

- Strain
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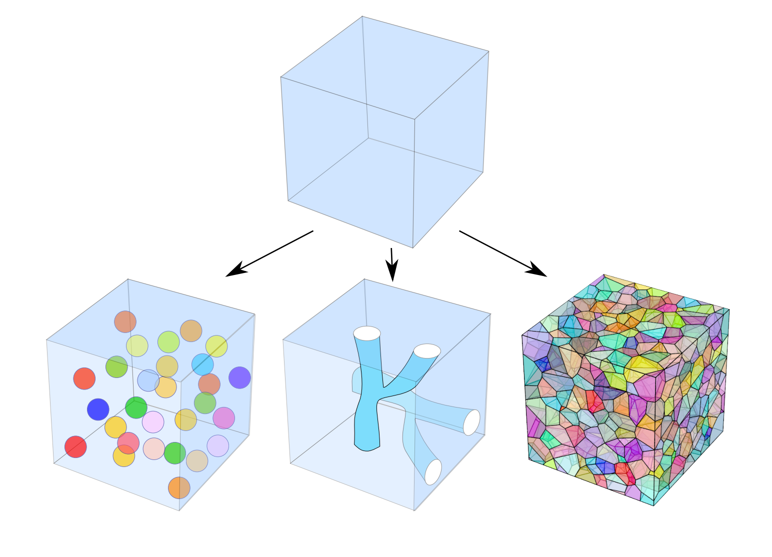
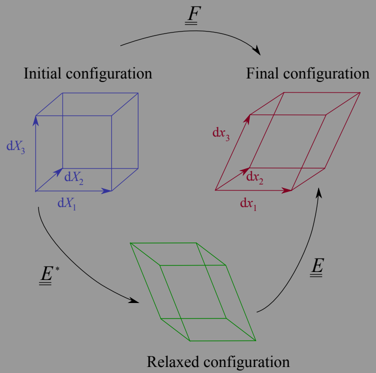
Eigenstrain

Phenomena arsing at a scale you did not model ?



Eigenstrain

Multiplicative decomposition



- $\underline{\underline{F}}$: transformation gradient, $\underline{\underline{E}}$ elastic tensor, $\underline{\underline{E}}^*$ eigenstrain

$$\boxed{\underline{\underline{F}} = \underline{\underline{E}}.\underline{\underline{E}}^*}$$

Eigenstrain

Transformation gradient

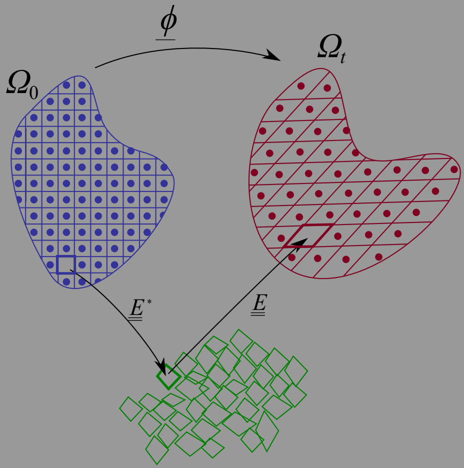
- $\underline{\underline{F}} = \underline{\underline{\nabla \phi}}$
- $\underline{\underline{E}}^*$ incompatible not associated to any transformation
- $\underline{\underline{E}}$ incompatible not associated to any transformation

Relaxed configuration

- Ω_0 initial domain
- Ω_t current domain
- There is no relaxed domain
- Relaxed configuration only defined at each material points

Eigenstrain

Transformation gradient



Not a domain !
Incompatibility

Eigenstrain

Examples

- Thermal expansion $\underline{\underline{E}}^* = \underline{\underline{1}} + \underline{\underline{\alpha}}\Delta T$
- Volume variation (phase transition) $\underline{\underline{E}}^* = \lambda\underline{\underline{1}}$
- Transformation induced plasticity
- Hygrometry (i.e., moisture content in wood)
- $\dots$

Continuum mechanics

- Strain
- Stress
- Behavior
- Eigenstrain
- Residual stresses
- Navier equation

Residual stresses

Misleading concept

- Eigenstrain $\underline{\underline{E}}^*$
- $\underline{\underline{F}} = \underline{\underline{E}}.\underline{\underline{E}}^*$
- $\underline{\underline{\sigma}} \sim \underline{\underline{E}}^T.\underline{\underline{E}}$

$$\underline{\underline{\sigma}} = \frac{\mu}{J^{\frac{5}{3}}}\underline{\underline{E}}.\underline{\underline{E}}^T + \left(k(J-1) - \frac{\mu}{J^{\frac{5}{3}}} \frac{\text{tr}\left(\underline{\underline{E}}.\underline{\underline{E}}^T\right)}{3} \right) \underline{\underline{1}}$$

- What does mean residual stress relaxation ?
  - Boundary conditions have changed (e.g., cuts) $\Rightarrow$ distortions
  - Eigenstrain evolution (e.g., grain growth etc.)

Residual stresses

Linearization

- Eigenstrain $\underline{\underline{\varepsilon}}^*$
- $\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^e + \underline{\underline{\varepsilon}}^*$
- $\underline{\underline{\sigma}} \sim \underline{\underline{\varepsilon}}^e$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}}^e = \underline{\underline{\underline{\underline{C}}}} : \left(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^*\right)$$

- Residual stresses due to elastic accommodation of the eigenstrain

Continuum mechanics

- Strain
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- Behavior
- Eigenstrain
- Residual stresses
- Navier equation

Navier equation

Strong equation set

- Equilibrium

$$\mathrm{div}\left[\underline{\underline{\sigma}}\right] = 0$$

- Compatibility

$$\underline{\underline{\varepsilon}} = \frac{1}{2}\left(\underline{\underline{\nabla u}} + \underline{\underline{\nabla u}}^T\right)$$

- Behavior

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}}^e = \underline{\underline{\underline{\underline{C}}}} : \left(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^*\right)$$

- Isotropic behavior

$$\underline{\underline{\sigma}} = \lambda \mathrm{tr}\left[\underline{\underline{\varepsilon}}^e\right]\underline{\underline{1}} + 2\mu\underline{\underline{\varepsilon}}^e$$

- Boundary conditions

$$\begin{cases} \forall \underline{x} \in \partial\Omega_T,\ \underline{\underline{\sigma}}(\underline{x}).\underline{n}(\underline{x}) = \underline{T}(\underline{x}) \\ \forall \underline{x} \in \partial\Omega_u,\ \underline{u}(\underline{x}) = \underline{u}^d(\underline{x}) \end{cases}$$

Navier equation

Strong equation set

- Equilibrium

$$\operatorname{div}\left[\lambda \operatorname{tr}\left[\underline{\underline{\varepsilon}}^e\right]\underline{\underline{1}} + 2\mu\underline{\underline{\varepsilon}}^e\right] = 0$$

- Eigenstrain as a right side term

$$\operatorname{div}\left[\lambda \operatorname{tr}\left[\underline{\underline{\varepsilon}}\right]\underline{\underline{1}} + 2\mu\underline{\underline{\varepsilon}}\right] = \operatorname{div}\left[\lambda \operatorname{tr}\left[\underline{\underline{\varepsilon}}^*\right]\underline{\underline{1}} + 2\mu\underline{\underline{\varepsilon}}^*\right]$$

- Navier equation

$$\boxed{\underline{\Delta u} + \frac{\lambda+\mu}{\mu}\underline{\nabla}\operatorname{div}\underline{u} = \operatorname{div}\left[\lambda \operatorname{tr}\left[\underline{\underline{\varepsilon}}^*\right]\underline{\underline{1}} + 2\mu\underline{\underline{\varepsilon}}^*\right]}$$

- Boundary conditions

$$\begin{cases} \forall \underline{x} \in \partial\Omega_T,\ \underline{\underline{\sigma}}(\underline{x}).\underline{n}(\underline{x}) = \underline{T}(\underline{x}) \\ \forall \underline{x} \in \partial\Omega_u,\ \underline{u}(\underline{x}) = \underline{u}^d(\underline{x}) \end{cases}$$

Navier equation

Eigenstrain : physical mechanisms

- Thermal expansion
- Volume variation (phase transition)
- Transformation induced plasticity
- Hygrometry (i.e., moisture content in wood)
- ...

Residual stresses : misleading concept

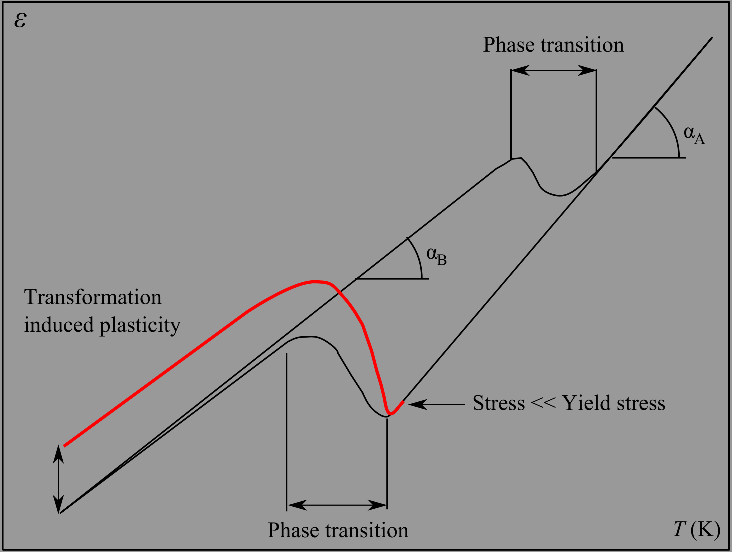
- Indirectly related to internal mechanisms
- Necessitates additional computation
- Depends on boundary conditions

Lecture outline

Transformation induced plasticity

- Experimental evidence
- Lower scale mechanims
- Simple modeling
- Experimental validation

Experimental evidence

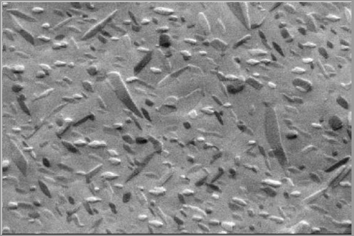


Transformation induced plasticity

- Experimental evidence
- Lower scale mechanims
- Simple modeling
- Experimental validation

Lower scale mechanims

Microscale



- Local plasticity
- Geometrical mismatch
- Plastic flow
- Preferential orientations

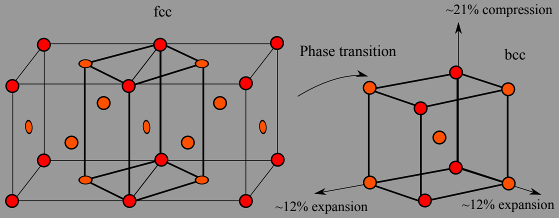
Macroscale

- Volume average
- Residual strain
- Global plasticity
- TRIP

Transformation induced plasticity

- Experimental evidence
- Lower scale mechanims
- Simple modeling
- Experimental validation

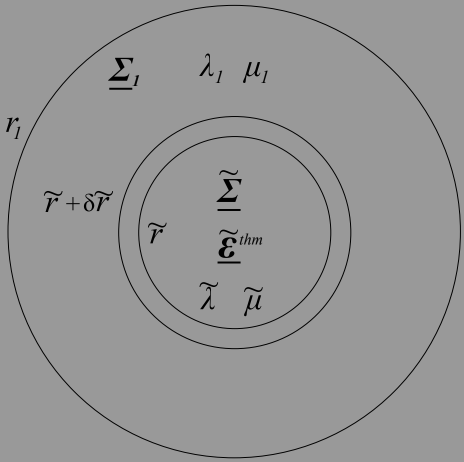
Simple modeling



Eigenstrain $\underline{\underline{\varepsilon}}^{thm}$

- Hydrostatic part : $\underline{\underline{\varepsilon}}^{thm,h} = \frac{\mathrm{tr}\left(\underline{\underline{\varepsilon}}^{thm}\right)}{3}\underline{\underline{1}}$
  - Volume variation, Density mismatch
  - **Not dependent on crystallographic directions**
- Deviatoric part : $\underline{\underline{\varepsilon}}^{thm,d} = \underline{\underline{\varepsilon}}^{thm} - \frac{\mathrm{tr}\left(\underline{\underline{\varepsilon}}^{thm}\right)}{3}\underline{\underline{1}}$
  - Very large but often neglected
  - Inclusions isotropically orientated
  - **Dependent on crystallographic directions**

Simple modeling



Simple modeling

Transformation induced plastic strain rate

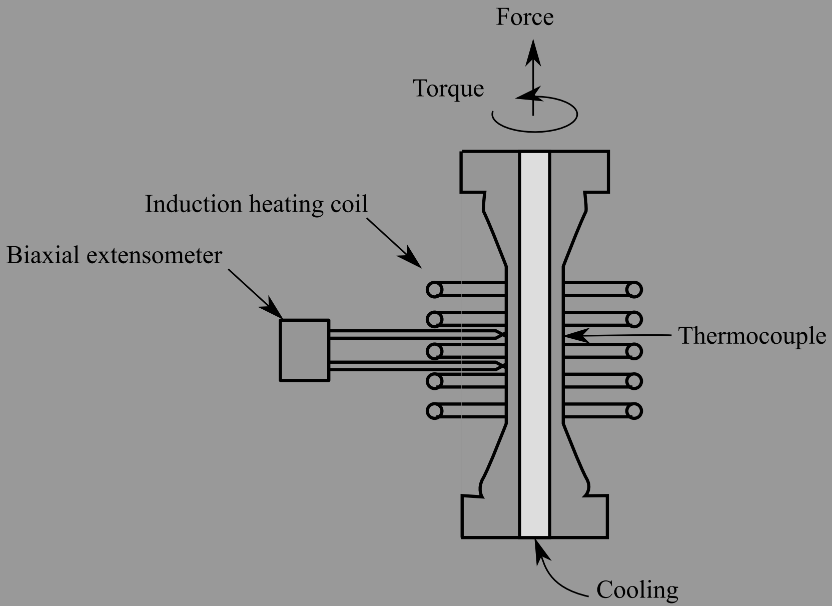
$$\underline{\underline{\dot{E}}}^{tp} = \sum_{p=2}^{N} \frac{\underline{\underline{S}}_p}{\sigma_p^Y} \frac{\sigma_p^Y - \Sigma_p^{eq}}{\mu_p \xi_p} \dot{X}_p + \begin{cases} 0 & \text{if} \quad \left|\widetilde{\varepsilon}^{thm}\right| < \dfrac{\Delta\sigma^Y}{\zeta} \\ -\dfrac{3\left|\widetilde{\varepsilon}^{thm}\right| \underline{\underline{S}}_1}{\sigma_1^Y} \ln\left(\dfrac{\Delta\sigma^Y}{\left|\widetilde{\varepsilon}^{thm}\right| \zeta}\right) \displaystyle\sum_{\substack{p=2 \\ \dot{X}_p > 0}}^{N} \dot{X}_p & \text{if} \quad \widetilde{X} \leq \dfrac{\Delta\sigma^Y}{\zeta \left|\widetilde{\varepsilon}^{thm}\right|} \leq 1 \\ -\dfrac{3\left|\widetilde{\varepsilon}^{thm}\right| \underline{\underline{S}}_1}{\sigma_1^Y} \ln\left(\widetilde{X}\right) \displaystyle\sum_{\substack{p=2 \\ \dot{X}_p > 0}}^{N} \dot{X}_p & \text{if} \quad \widetilde{X} > \dfrac{\Delta\sigma^Y}{\zeta \left|\widetilde{\varepsilon}^{thm}\right|} \end{cases}$$

- X_p phase proportion of p-th phase and $\widetilde{X} = \sum_{p=2}^{N} X_p$
- $\sigma_p^Y(T)$ yield stress of p-th phase
- $\underline{\underline{S}}_p$ average stress deviator in the p-th phase
- $\widetilde{\varepsilon}^{thm}$ average volume variation in all product phases
- ζ, ξ_p material parameters

Transformation induced plasticity

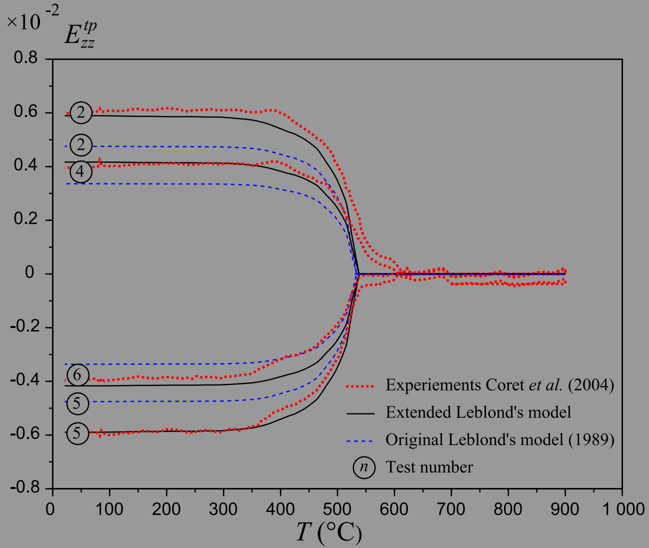
- Experimental evidence
- Lower scale mechanims
- Simple modeling
- Experimental validation

Experimental validation



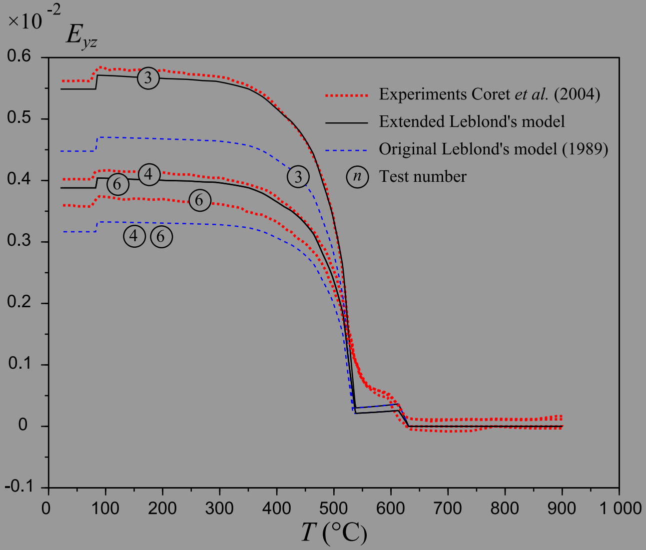
Experimental validation

Experimental validation (Coret et al. 2004)



Experimental validation

Experimental validation (Coret et al. 2004)



Lecture outline

Application to additive manufacturing

- Modeling residual stresses
- Measuring residual stresses
- Large scale processes

Modeling residual stresses

Decoupling

- Thermal analysis of the process coupled with phase transitions

$$\mathrm{div}(\lambda(T)\underline{\nabla} T) - \rho c_p(T)\frac{\partial T}{\partial t} = -\sum_{\phi=1}^{N_\phi} \Delta H_\phi(T)\dot{X}_\phi$$

- Important Compute eigenstrain $\underline{\underline{\varepsilon}}^*$
  - Thermal expansion
  - Volume variation due to phase transitions
  - Transformation induced plasticity
- Solve the elastic-plastic mechanical problem

$$\underline{\Delta u} + \frac{\lambda+\mu}{\mu}\underline{\nabla}\mathrm{div}\,\underline{u} = \mathrm{div}\left[\lambda \mathrm{tr}\left[\underline{\underline{\varepsilon}}^* + \underline{\underline{\varepsilon}}^p\right]\underline{\underline{1}} + 2\mu\left(\underline{\underline{\varepsilon}}^* + \underline{\underline{\varepsilon}}^p\right)\right]$$

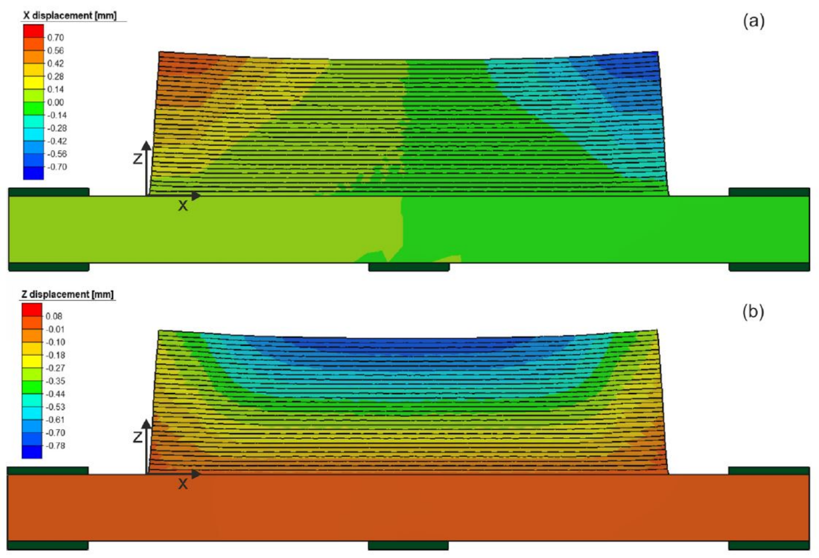
Modeling residual stresses

Fully coupled

- Solve simultaneously
  - Thermal analysis
  - Eigenstrain
  - Mechanical problem and displacements
- Computationally costly

Modeling residual stresses

Biegler et al. 2018

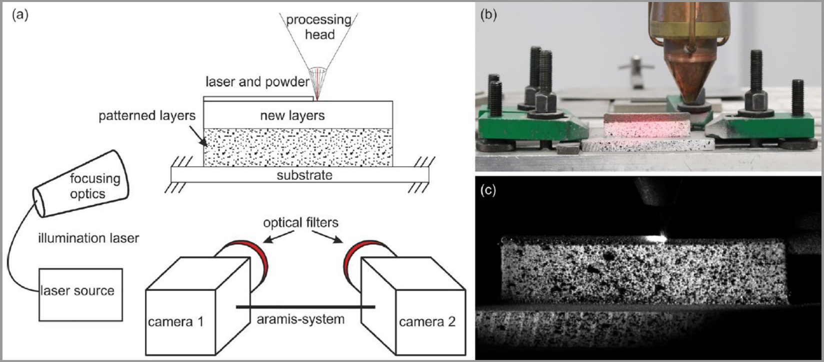


Application to additive manufacturing

- Modeling residual stresses
- Measuring residual stresses
- Large scale processes

Measuring residual stresses

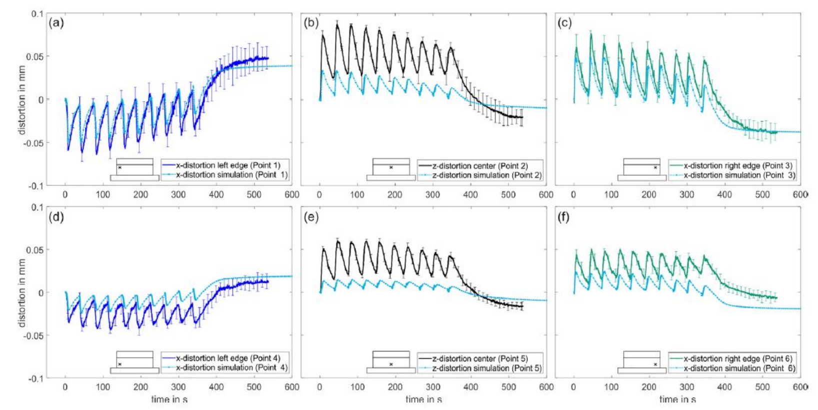
Biegler et al. 2018



- Displacement measurement
- During the process
- After cutting

Measuring residual stresses

Biegler et al. 2018



Measuring residual stresses

Measuring residual stresses

Measuring residual stresses

Direct method

- X-Ray Diffraction
- Stresses affect the crystal lattice
- Strain gauge
- Local measurements
  - Several measurement points
  - Average

Application to additive manufacturing

- Modeling residual stresses
- Measuring residual stresses
- Large scale processes

Large scale processes

Multiscale problem

- Eigenstrain results from lower scale phenomena
  - Dilatation of the crystal lattice : thermal expansion
  - Chemical reactions in the microstructure
  - Local plastic deformation
  - Fluid flow in the microstructure
  - ...
- Stresses depends on the entire structure
- Reciprocal interactions between scales

Need for multiscale approaches with limited computational cost