Operations research in the railway industry

Guillaume Dalle

ENPC - CERMICS

20/05/2020

About me

About me

- Former student of M2 *Mathématiques, Vision, Apprentissage* with a bit of *Master Parisien de Recherche Opérationnelle*

About me

- Former student of M2 *Mathématiques, Vision, Apprentissage* with a bit of *Master Parisien de Recherche Opérationnelle*

- Now first-year PhD student at CERMICS
  Supervisors: Axel Parmentier and Yohann de Castro

About me

Former student of M2 *Mathématiques, Vision, Apprentissage* with a bit of *Master Parisien de Recherche Opérationnelle*

Now first-year PhD student at CERMICS
Supervisors: Axel Parmentier and Yohann de Castro

Working with SNCF on optimizing TGV maintenance:

- Predicting future failures
- Assigning rolling stock to trips
- Organizing maintenance depot operations

About me

Former student of M2 *Mathématiques, Vision, Apprentissage* with a bit of *Master Parisien de Recherche Opérationnelle*

Now first-year PhD student at CERMICS
Supervisors: Axel Parmentier and Yohann de Castro

Working with SNCF on optimizing TGV maintenance:

- Predicting future failures
- Assigning rolling stock to trips
- Organizing maintenance depot operations

Feel free to contact me at guillaume.dalle@enpc.fr

Table of Contents

Table of Contents

Decision-making in the railway industry

Decision-making in the railway industry

- Various temporal and geographic scales

Decision-making in the railway industry

- Various temporal and geographic scales
- Multiple actors and resources, endless constraints

Decision-making in the railway industry

- Various temporal and geographic scales
- Multiple actors and resources, endless constraints
- One objective: good service at reasonable cost

Decision-making in the railway industry

- Various temporal and geographic scales
- Multiple actors and resources, endless constraints
- One objective: good service at reasonable cost

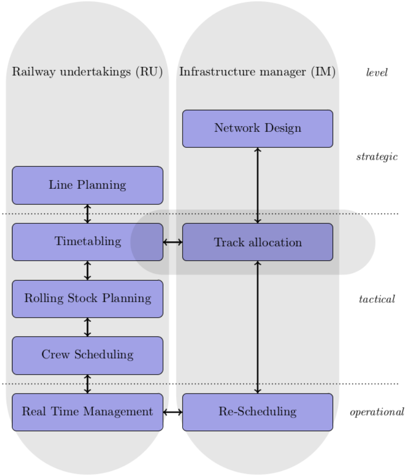


Figure 1: Planning process [Schlechte, 2012]

Why should we optimize anything?

Why should we optimize anything?

🕗 To improve passenger satisfaction

Why should we optimize anything?

- To improve passenger satisfaction
- To reduce costs and resource use

Why should we optimize anything?

- To improve passenger satisfaction
- To reduce costs and resource use
- To focus skilled operators on the truly difficult issues

Why should we optimize anything?

- To improve passenger satisfaction
- To reduce costs and resource use
- To focus skilled operators on the truly difficult issues

WARNING

Optimizing is not always necessary or useful.
Human expertise remains essential.

Challenges for railway optimization

Challenges for railway optimization

✈️ Mathematical complexity > airline industry

- More moving stuff
- Less moving freedom

Challenges for railway optimization

Mathematical complexity $>$ airline industry

- More moving stuff
- Less moving freedom

Complex information systems

Challenges for railway optimization

Mathematical complexity > airline industry

- More moving stuff
- Less moving freedom

Complex information systems

Until recently, lack of incentives

Table of Contents

Goal

Everything in this section is based on Zwaneveld et al. [1996].

Goal

Everything in this section is based on Zwaneveld et al. [1996].

Input Arrival and departure times at a station

Goal

Everything in this section is based on Zwaneveld et al. [1996].

Input Arrival and departure times at a station

Output Optimal "routing" of all trains

Goal

Everything in this section is based on Zwaneveld et al. [1996].

Input Arrival and departure times at a station

Output Optimal "routing" of all trains

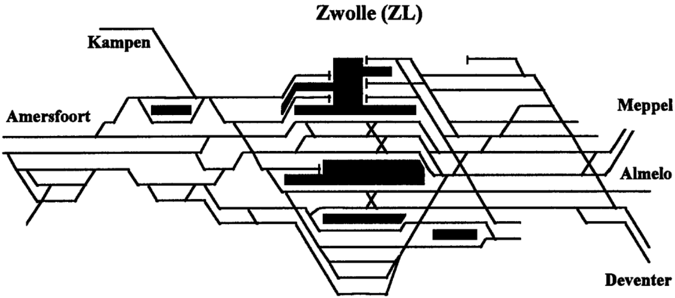


Figure 2: The railway station of Zwolle

Variables

Variables

Train routes = sequences of track sections from station entrance to exit

Variables

Train routes = sequences of track sections from station entrance to exit

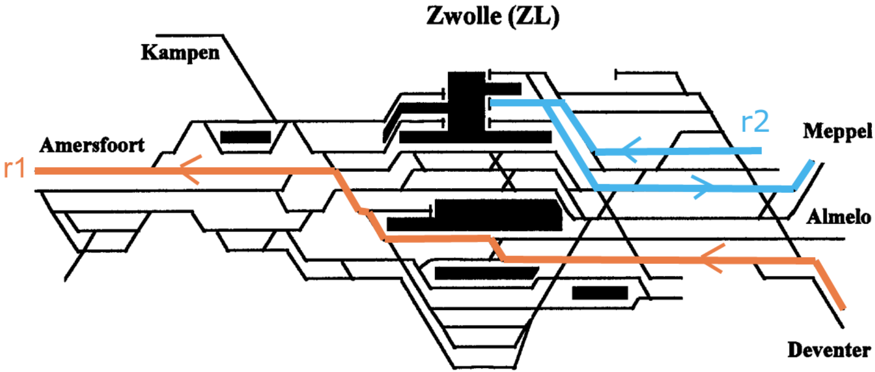


Figure 3: Two possible routes through Zwolle station

Constraints (1)

Constraints (1)

Individual constraints: each train can use specific routes & platforms

Constraints (1)

Individual constraints: each train can use specific routes & platforms

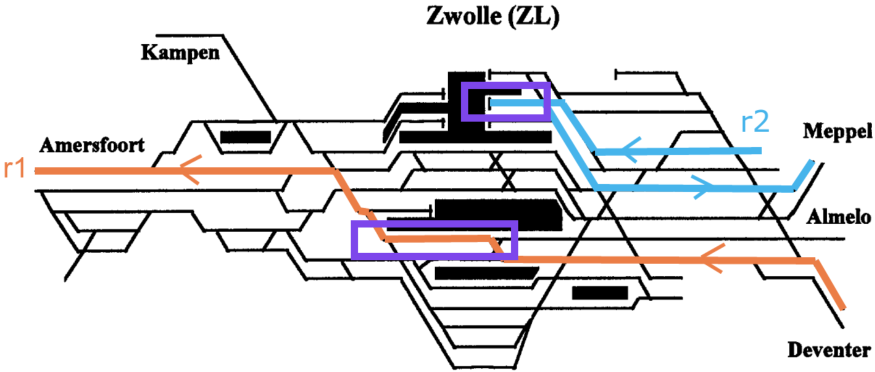


Figure 4: Two routes and their platforms

Constraints (2)

Constraints (2)

Couple constraints: two simultaneous trains must have compatible routes

Constraints (2)

Couple constraints: two simultaneous trains must have compatible routes

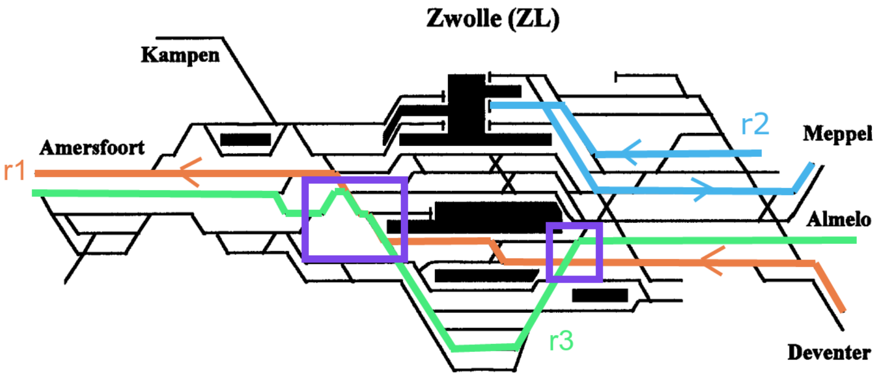


Figure 5: Conflicting routes

Some notations

Some notations

- $t \in T$: trains
- $r \in R$: routes

Some notations

- $t \in T$: trains
- $r \in R$: routes
- R_t: routes possible for train t

Some notations

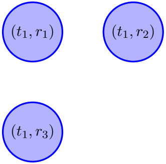
- $t \in T$: trains
- $r \in R$: routes
- R_t: routes possible for train t
- $IC_{t,t'}$: couples of incompatible routes for t and t'

Some notations

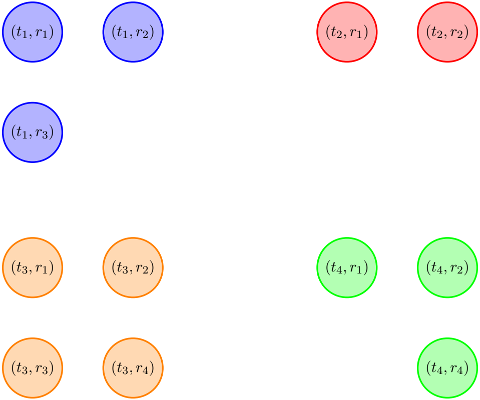
- $t \in T$: trains
- $r \in R$: routes
- R_t: routes possible for train t
- $IC_{t,t'}$: couples of incompatible routes for t and t'
- $X_{t,r} = 1$ iff train t is assigned to route r

A maximum independent set problem

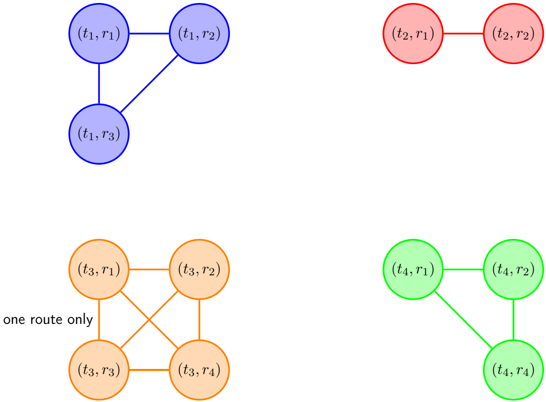
A maximum independent set problem



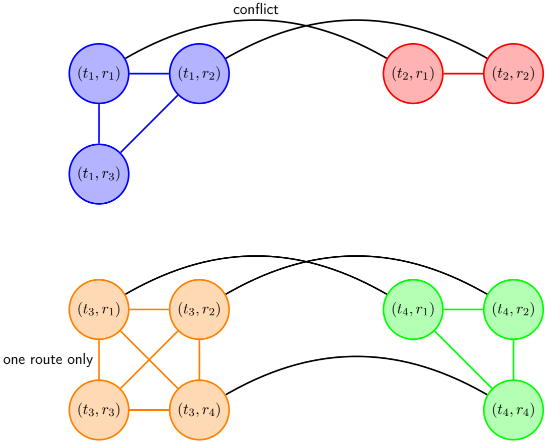
A maximum independent set problem



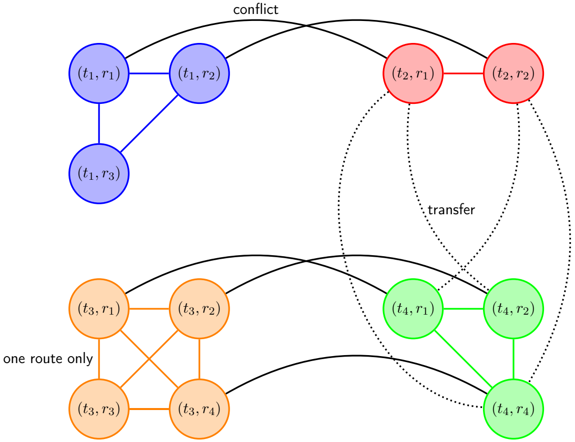
A maximum independent set problem



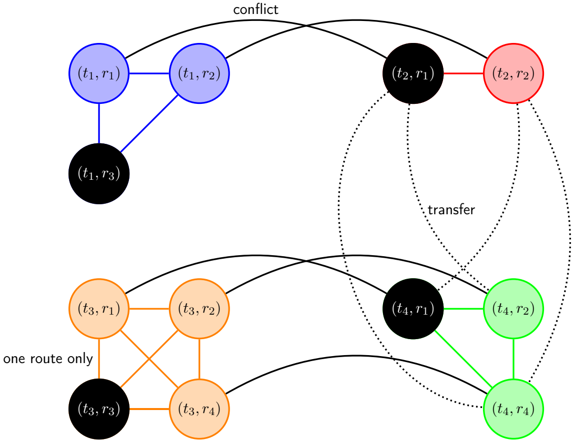
A maximum independent set problem



A maximum independent set problem



A maximum independent set problem



A maximum independent set problem (2)

A maximum independent set problem (2)

- We build the undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with

$$\mathcal{V} = \{(t, r) : t \in T, r \in R_t\}$$
$$\mathcal{E} = \{((t, r), (t', r')) : t = t' \text{ or } (r, r') \in IC_{t,t'}\}$$

A maximum independent set problem (2)

- We build the undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with

$$\mathcal{V} = \{(t, r) : t \in T, r \in R_t\}$$
$$\mathcal{E} = \{((t, r), (t', r')) : t = t' \text{ or } (r, r') \in IC_{t,t'}\}$$

- We look for the largest set of mutually unrelated vertices.

A maximum independent set problem (2)

- We build the undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with

$$\mathcal{V} = \{(t, r) : t \in T, r \in R_t\}$$
$$\mathcal{E} = \{((t, r), (t', r')) : t = t' \text{ or } (r, r') \in IC_{t,t'}\}$$

- We look for the largest set of mutually unrelated vertices.
- If there is a feasible platforming, it will have $|T|$ elements.

An Integer Linear Program (1)

An Integer Linear Program (1)

- This problem can be formulated as an Integer Linear Program.

An Integer Linear Program (1)

- This problem can be formulated as an Integer Linear Program.
- The decision variables are $X_{t,r} = 1$ iff train t is assigned to route r

An Integer Linear Program (1)

- This problem can be formulated as an Integer Linear Program.
- The decision variables are $X_{t,r} = 1$ iff train t is assigned to route r
- The objective is to maximize the number of trains assigned:

$$\max \sum_{t \in T} \mathbf{1}[t \text{ is assigned}] = \max \sum_{t \in T} \sum_{r \in R_t} X_{t,r}$$

An Integer Linear Program (2)

An Integer Linear Program (2)

What are the constraints ?

An Integer Linear Program (2)

What are the constraints ?

- Either we select a route or we don't: $X_{t,r} \in \{0, 1\}$

An Integer Linear Program (2)

What are the constraints ?

- Either we select a route or we don't: $X_{t,r} \in \{0, 1\}$
- We cannot select two different routes r and r' for the same train t.

An Integer Linear Program (2)

What are the constraints ?

- Either we select a route or we don't: $X_{t,r} \in \{0, 1\}$
- We cannot select two different routes r and r' for the same train t. Hence, at most one of $X_{t,r}$ and $X_{t,r'}$ is equal to 1.

An Integer Linear Program (2)

What are the constraints ?

- Either we select a route or we don't: $X_{t,r} \in \{0, 1\}$
- We cannot select two different routes r and r' for the same train t. Hence, at most one of $X_{t,r}$ and $X_{t,r'}$ is equal to 1. This means:

$$\forall (r, r') \in R_t^2, r \neq r', \quad X_{t,r} + X_{t,r'} \leq 1$$

An Integer Linear Program (2)

What are the constraints ?

- Either we select a route or we don't: $X_{t,r} \in \{0, 1\}$

- We cannot select two different routes r and r' for the same train t. Hence, at most one of $X_{t,r}$ and $X_{t,r'}$ is equal to 1. This means:

$$\forall (r, r') \in R_t^2, r \neq r', \quad X_{t,r} + X_{t,r'} \leq 1$$

- We cannot select incompatible routes r and r' for two trains t and t':

$$\forall (r, r') \in IC_{t,t'}, \quad X_{t,r} + X_{t',r'} \leq 1$$

An integer linear program (3)

An integer linear program (3)

We finally obtain:

An integer linear program (3)

We finally obtain:

$$\max \sum_{t \in T} \sum_{r \in R_t} X_{t,r}$$

$$\text{s.t.} \left\{ \right.$$

An integer linear program (3)

We finally obtain:

$$\max \sum_{t \in T} \sum_{r \in R_t} X_{t,r}$$

$$\text{s.t.} \begin{cases} X_{t,r} \in \{0,1\} & \forall t \in T, \forall r \in R_t \\ & \end{cases}$$

An integer linear program (3)

We finally obtain:

$$\max \sum_{t \in T} \sum_{r \in R_t} X_{t,r}$$

$$\text{s.t.} \begin{cases} X_{t,r} \in \{0,1\} & \forall t \in T, \forall r \in R_t \\ X_{t,r} + X_{t,r'} \leq 1 & \forall t \in T, \forall (r,r') \in R_t^2, r \neq r' \\ \end{cases}$$

An integer linear program (3)

We finally obtain:

$$\max \sum_{t \in T} \sum_{r \in R_t} X_{t,r}$$

$$\text{s.t.} \begin{cases} X_{t,r} \in \{0,1\} & \forall t \in T, \forall r \in R_t \\ X_{t,r} + X_{t,r'} \leq 1 & \forall t \in T, \forall (r,r') \in R_t^2, r \neq r' \\ X_{t,r} + X_{t',r'} \leq 1 & \forall (t,t') \in T^2, \forall (r,r') \in IC_{t,t'} \end{cases}$$

Clever initialization

Clever initialization

- Initial assignment: until all trains are routed

Clever initialization

- Initial assignment: until all trains are routed
  1. Select the train t^* with the fewest available routes

Clever initialization

- Initial assignment: until all trains are routed
  1. Select the train t^* with the fewest available routes
  2. Choose a route r^* for t^* that "leaves many options" to the other trains

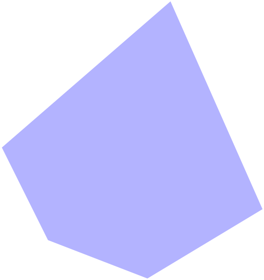
Clever initialization

- Initial assignment: until all trains are routed
  1. Select the train t^* with the fewest available routes
  2. Choose a route r^* for t^* that "leaves many options" to the other trains

- Feasible set preprocessing: remove dominated routings

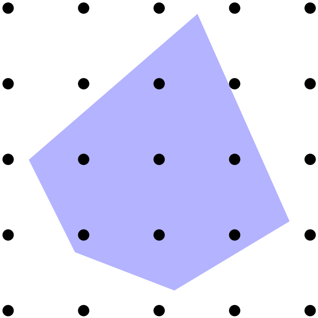
Valid inequalities

Valid inequalities



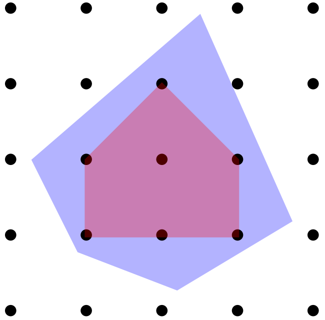
- Linear Program = polyhedron

Valid inequalities



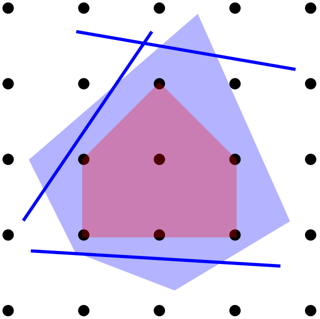
- Linear Program = polyhedron
- Integer Linear Program = discrete points inside a polyhedron

Valid inequalities



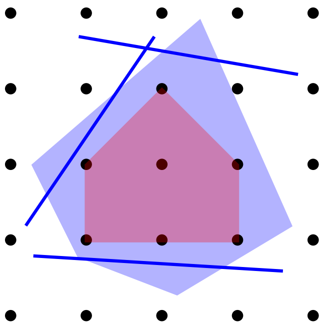
- Linear Program = polyhedron
- Integer Linear Program = discrete points inside a polyhedron
- Tighter polyhedron $\implies$ faster branch & bound

Valid inequalities



- Linear Program = polyhedron
- Integer Linear Program = discrete points inside a polyhedron
- Tighter polyhedron $\Longrightarrow$ faster branch & bound
  Solution: add valid inequalities

Valid inequalities



- Linear Program = polyhedron
- Integer Linear Program = discrete points inside a polyhedron
- Tighter polyhedron $\implies$ faster branch & bound
  Solution: add valid inequalities

Example: clique inequalities for the maximum independent set

$$X_{t,r} + X_{t,r'} \leq 1 \quad \text{can be strengthened into} \quad \sum_{r \in R_t} X_{t,r} \leq 1$$

If you want to know more

If you want to know more

- Some reviews on railway optimization: Caprara et al. [2007, 2011]

If you want to know more

- Some reviews on railway optimization: Caprara et al. [2007, 2011]
- A recent book with many different problems: Borndörfer et al. [2018]

If you want to know more

- Some reviews on railway optimization: Caprara et al. [2007, 2011]
- A recent book with many different problems: Borndörfer et al. [2018]
- A master's thesis by a friend from SNCF on train platforming: Kamenga [2016]

References I

R. Borndörfer, T. Klug, L. Lamorgese, C. Mannino, M. Reuther, and T. Schlechte, editors. **Handbook of Optimization in the Railway Industry**. International Series in Operations Research & Management Science. Springer International Publishing, 2018. ISBN 978-3-319-72152-1. doi: 10.1007/978-3-319-72153-8.

A. Caprara, L. Kroon, M. Monaci, M. Peeters, and P. Toth. Chapter 3 Passenger Railway Optimization. In C. Barnhart and G. Laporte, editors, **Handbooks in Operations Research and Management Science**, volume 14 of **Transportation**, pages 129–187. Elsevier, Jan. 2007. doi: 10.1016/S0927-0507(06)14003-7.

A. Caprara, L. Kroon, and P. Toth. Optimization Problems in Passenger Railway Systems. In **Wiley Encyclopedia of Operations Research and Management Science**. American Cancer Society, 2011. ISBN 978-0-470-40053-1. doi: 10.1002/9780470400531.eorms0647.

F. Kamenga. **Gestion de l'occupation de Voie Des Gares En Temps Réel**. Mémoire de Master, Université Paris Dauphine - Mines ParisTech, 2016.

T. Schlechte. **Railway Track Allocation: Models and Algorithms**. PhD thesis, Technische Universität Berlin, 2012.

P. J. Zwaneveld, L. G. Kroon, H. E. Romeijn, M. Salomon, S. Dauzère-Pérès, S. P. M. Van Hoesel, and H. W. Ambergen. Routing Trains Through Railway Stations: Model Formulation and Algorithms. **Transportation Science**, 30(3):181–194, Aug. 1996. ISSN 0041-1655. doi: 10.1287/trsc.30.3.181.